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Modeling the interfacial effect on the yield strength and flow stress of thin metal films on substrates

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Abstract

It is shown in this paper that interfacial effects have a profound impact on the scale-dependent yield strength and strain hardening rates (flow stress) of metallic thin films on elastic substrates. This is achieved by developing a higher-order strain gradient plasticity theory based on the principle of virtual power and the laws of thermodynamics. This theory enforces microscopic boundary conditions at interfaces which relate a microtraction stress to the interfacial energy at the interface. It is shown that the film bulk length scale controls the size effect if a rigid interface is assumed whereas the interfacial length scale dominates if a compliant interface is assumed.

Keywords: Interfacial energy; Nonlocal; Size effect; Thin films; Length scale

1. Introduction

Accurate identification of the mechanical properties of metallic thin film materials is essential for the design, performance, and development of micro/nanoelectronics and micro/nanoelectromechanical systems (MEMS/NEMS) to be used, for example, as actuators or sensors (e.g. pressure, inertial, thermal, and chemical sensors, position detectors, accelerometers, magnetometers, micromirrors, etc.) The mechanical properties of thin films are different from those of the conventional or bulk counterparts because they are very sensitive to the microstructural features of the material such as the grain size, the finite number of grains through the thickness, surface and interface thickness, texture, and dislocation structure. Therefore, when one or more of the dimensions of a thin film begin to approach that of the film's microstructural features, the material mechanical properties (e.g. yield strength, strain hardening, fracture toughness) begin to exhibit a dependence on the structure. There are several pioneering studies that have experimentally identified the existence of size effects in single crystal and polycrystalline thin metal films (e.g. Stolken and Evans, 1998; Huang and Spaepen, 2000; Haque and Saif, 2003; Shrotriya et al., 2003; Espinosa et al., 2004).

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Plastic deformation in small-scale structures, accommodated by dislocation nucleation and movement, is strongly affected by interfaces. Until now, little attention is devoted to interfacial effect of the film-substrate interface on the scale-dependent plasticity in thin films. Investigations of interface conditions have recently been presented by Gudmundson (2004), Gurtin and Needleman (2005), Aifantis and Willis (2005), Abu Al-Rub et al. (2007). Interfaces between distinct regions can be locations for dislocations' blocking and pileups, and hence decreasing plastic flow and increasing the strength. This type of size effect could not be explained by the classical continuum mechanics since no length scale enters the constitutive description (Gao et al., 1999). Also, a lower-order gradient plasticity theory in the form as, for example, in Aifantis (1984), Acharya and Bassani (2000), Abu Al-Rub and Voyiadjis (2006) could not predict any boundary layer effect, which makes them unsuitable for modeling the interfacial effect in thin films. A higher-order gradient plasticity theory of the form presented in this paper should be adapted in modeling the film-substrate interface effect on scale-dependent plasticity in thin films. This is achieved by developing a higher-order strain gradient plasticity theory based on the principle of virtual power and nonlocal laws of thermodynamics that incorporate an interfacial energy term in the internal power that depends on the plastic strain state at the interface of the plastically deforming material. As a result, two material length scales are incorporated: one for the bulk material and one for the interface, which together control the size effect in thin films. The focus of this paper is on the role of film-substrate interfaces on the yield strength (i.e. onset of plasticity) and the flow stress (i.e. strain hardening rate).

2. Thermodynamics of higher-order gradient plasticity

The role of interfaces on constraining the plastic flow and then on the observed size effect in thin films can be incorporated by formulating a higher-order gradient plasticity theory. The classical (local) plasticity theory fails to predict any size effect due to the absence of a microstructural length scale parameter in its constitutive description whereas the gradient plasticity theory incorporates higher-order gradients of microstructural internal state variables and hence posses an intrinsic material length scale. In the following, this higher-order theory is formulated based on the principle of virtual power and the nonlocal laws of thermodynamics.

In the classical plasticity theory, the rate of the plastic strain tensor, $\dot{\varepsilon}^p_{ij} = \dot{p}N_{ij}$, where N_{ij} is the direction of the plastic flow, and the rate of the effective plastic strain variable, $\dot{p} = \|\dot{\varepsilon}^p_{ij}\| = \sqrt{\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij}}$, enter the definition of the internal virtual power besides the rate of the elastic strain tensor, $\dot{\varepsilon}^e_{ij} = \dot{\varepsilon}_{ij} - \dot{\varepsilon}^p_{ij}$, where $\dot{\varepsilon}_{ij}$ is the rate of the total strain tensor. However, due to the heterogeneous distribution of stationary microstructural features (e.g. grain/particle/aggregate size and distribution, grain boundaries, interfaces, etc) and deformation-dependent microstructural features (e.g. defects such as dislocations, micro-cracks, and micro-voids, twins, etc), gradients of the aforementioned internal state variables do exist and should be taken into consideration especially in small-scale systems (e.g. thin films, nano-wires, nano-composites, etc). This implies that the stress state at a material point depends on the strain at that point and the strains at the surrounding points (i.e. nonlocality). Since the focus of this paper is on the role of higher-order gradient plasticity on the scale-dependent behavior of thin metal films, higher-order gradient elasticity (i.e. gradients of the elastic strain tensor) will not be considered. Therefore, the internal virtual power depends not only on $\dot{\varepsilon}^e_{ij}$, $\dot{p}$, and $\dot{\varepsilon}^p_{ij}$, but also on the spatial higher-order gradients $\dot{\varepsilon}^p_{ij,k}$ and $\dot{p}_k$. The third-order tensor $\dot{\varepsilon}^p_{ij,k}$ introduces anisotropy through kinematic hardening which is attributed to the net Burgers vector being not equal to zero at the microscale (Nye, 1953). The first-order gradient $\dot{p}_{,k}$ introduces isotropic hardening or internal history which is attributed to the accumulation of the so-called geometrically necessary dislocations (Ashby, 1970). The plastic strain gradient, $\varepsilon^p_{ij,k}$, is related to the geometrically necessary dislocation density tensor, G_{ij}, through the following relation (Nye, 1953):

$$G_{ij} = e_{irq}\varepsilon^p_{jq,\mathrm{r}} \tag{1}$$

where e_{irq} is the permutation tensor. In addition, the gradient of the effective plastic strain, $p_{,k}$, is related to the effective density of geometrically necessary dislocation, ρ_G, through the following relation (Ashby, 1970):

$$\rho_G = \frac{\sqrt{p_{,k}p_{,k}}}{b} \tag{2}$$

where b is the magnitude of the Burgers vector. Therefore, the presence of higher-order gradients through the rate of plastic strain tensor (i.e. $\dot{\varepsilon}^p_{ij,k}$) leads to higher-order gradients in the rate of accumulation of plastic strain (i.e. $\dot{p}_{,k}$) such that one cannot exist without the other. Therefore, in order to complete the description of higher-order gradient plasticity, the total rate of accumulation of the plastic strain gradients should also be considered in formulating the internal virtual power, such that:

$$\dot{e} = \sqrt{\dot{\varepsilon}^p_{ij,k}\dot{\varepsilon}^p_{ij,k}} = \sqrt{\dot{p}_{,k}\dot{p}_{,k}} \qquad \text{with} \qquad e = \int_0^t \dot{e}\,\mathrm{d}t \tag{3}$$

should also be considered in the constitutive description as it introduces additional isotropic hardening. In obtaining the equality in Eq. $(3)_1$, the gradient of the plastic strain direction, $N_{ij,k}$, is neglected. This assumption is supported by the localization of plastic deformation such that the plastic flow direction is almost the same within the localized zone. Moreover, adopting this assumption greatly simplifies the subsequent derivations. Therefore, the generalized rate of total accumulation of the plastic strain and plastic strain gradients can be defined as (e.g. Fleck and Hutchinson, 2001; Gurtin, 2003; Gudmundson, 2004):

$$\dot{E}^2 = \dot{p}^2 + \ell^2\dot{e}^2 = \dot{p}^2 + \ell^2\dot{p}_{,k}\dot{p}_{,k} = \dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij} + \ell^2\dot{\varepsilon}^p_{ij,k}\dot{\varepsilon}^p_{ij,k} \tag{4}$$

where ℓ is the material length scale parameter used for dimensional consistency and $E = \int_0^t \dot{E}\mathrm{d}t$ is the nonlocal effective plastic strain, which is intended to measure the total dislocation density. Moreover, the physical argument of Eq. (4) is that E provides an overall scalar measure of the density of dislocations, with p giving a measure of statistically stored dislocation density and e providing a measure of the geometrically necessary dislocation density. With the absence of plastic strain gradients, E reduces to the local effective plastic strain p. Hence, the author believes that for a complete constitutive description at small length scales, the internal power and the Helmholtz free energy should not include only the effects of $\dot{\varepsilon}^p_{ij}$ and $\dot{p}$ but should also include the effects of $\dot{\varepsilon}^p_{ij,k}$,$\dot{p}_{,k}$, and $\dot{e} = \|\dot{\varepsilon}^p_{ij,k}\| = \|\dot{p}_{,k}\|$. Although these variables may have a common origin in dislocation storage and motion, they will be treated independent of each other. This gives different physical interpretations that guide one to different evolution equations and allowing one to computationally introduce the influence of one scale on the other (e.g. the effect of mesoscale on macroscale). For example, dislocation interactions are observed on a mesolevel with length scale of $0.1 - 10$ μm affecting strongly the material behavior on the macrolevel with length scale $\geqslant 100$ μm. However, those variables are considered here mathematically related to their local counterparts and, therefore, special care must be taken to properly account for their coupling. For example, similar to the definition of direction of plastic strain, $N_{ij} = \dot{\varepsilon}^p_{ij}/\dot{p}$, one can define the directions of the plastic strain gradient, $M_{ijk} = \dot{\varepsilon}^p_{ij,k}/\dot{e}$, and the gradient of the effective plastic strain, $\mathbb{N}_k = \dot{p}_{,k}/\dot{e}$, such that the following relations can be written

$$\dot{p} = \dot{\varepsilon}^p_{ij}N_{ij}, \qquad \dot{p}_{,k} = \dot{\varepsilon}^p_{ij,k}N_{ij}, \qquad \dot{e} = \dot{\varepsilon}^p_{ij,k}M_{ijk}, \qquad \dot{e} = \dot{\varepsilon}^p_{ij,k}N_{ij}\mathbb{N}_k \tag{5}$$

2.1. Principle of virtual power

The principle of virtual power is the assertion that, given any sub-body Γ, the virtual power expended on Γ by materials or bodies exterior to Γ (i.e. external power) be equal to the virtual power expended within Γ (i.e. internal power). Let n_i denotes the outward unit normal to $\partial\Gamma$. The external expenditure of power is assumed to arise from a macroscopic surface traction t_i, the microtraction stress tensor, m_{ij}, conjugate to $\dot{\varepsilon}^p_{ij}$, defined for each unit vector n_i normal on the boundary $\partial\Gamma$ of Γ. Therefore, by neglecting body forces, one can write the external virtual power for static problems in the following form

$$P_{\text{ext}} = \int_{\partial\Gamma} \left(t_i\delta v_i + m_{ij}\delta\dot{\varepsilon}^p_{ij}\right)\mathrm{d}A \tag{6}$$

The kinematical fields δv_i and $\delta\dot{\varepsilon}^p_{ij}$ are considered here as virtual, where δ is the variation parameter and v_i is the velocity vector.

The external power is balanced by an internal expenditure of power characterized by the Cauchy stress tensor σ_{ij} defined over Γ for all time, the backstress X_{ij} associated with kinematic hardening, and the drag-stress R associated with isotropic hardening. However, since the goal of this paper is a theory that incorporates the gradients of the plastic strain, one also considers power expenditures associated with kinematic variables $\dot{\varepsilon}^p_{ij,k}$, $\dot{p}_{,k}$, and $\dot{e}$. One, therefore, can assume that additional power is expended internally by the higher-order microstress S_{ij} conjugate to $\dot{\varepsilon}^p_{ij,k}$, the higher-order microforce vector Q_k conjugate to $\dot{p}_{,k}$, and the conjugate force K which is meant to account for the additional isotropic gradient hardening represented by $\dot{e}$. Specifically, the internal virtual power is assumed to have the following form

$$P_{\text{int}} = \int_\Gamma \left(\sigma_{ij}\delta\dot{\varepsilon}^e_{ij} + X_{ij}\delta\dot{\varepsilon}^p_{ij} + R\delta\dot{p} + S_{ijk}\delta\dot{\varepsilon}^p_{ij,k} + Q_k\delta\dot{p}_{,k} + K\delta\dot{e}\right)\mathrm{d}V \tag{7}$$

and to balance P_{ext}, Eq. (6), in the sense that $P_{\text{ext}} = P_{\text{int}}$.

Substituting Eqs. (5)–(7) along with $\dot{\varepsilon}^e_{ij} = \dot{\varepsilon}_{ij} - \dot{\varepsilon}^p_{ij}$ into the virtual power balance, $P_{\text{ext}} = P_{\text{int}}$, and then applying the divergence theorem yields, after some lengthy manipulations, the following result:

$$\int_\Gamma \sigma_{ij,j}\delta v_i \mathrm{d}V + \int_{\partial\Gamma}(t_i - \sigma_{ij}n_j)\delta v_i \mathrm{d}A + \int_\Gamma [\tau_{ij} - X_{ij} + S_{ijk,k} - (R - Q_{k,k} - (K\mathbb{N}_k)_{,k})N_{ij}]\delta\dot{\varepsilon}^p_{ij}\mathrm{d}V$$
$$+ \int_{\partial\Gamma}[m_{ij} - (S_{ijk} + (Q_k + K\mathbb{N}_k)N_{ij})n_k]\delta\dot{\varepsilon}^p_{ij}\mathrm{d}A = 0 \tag{8}$$

where τ_{ij} is the deviatoric part of σ_{ij} (i.e. $\tau_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$). The fields Γ,δv_i, and $\delta\dot{\varepsilon}^p_{ij}$ may be arbitrarily specified if and only if

$$\sigma_{ij,j} = 0, \qquad t_i = \sigma_{ij}n_j \tag{9}$$
$$\tau_{ij} - X_{ij} + S_{ijk,k} - [R - Q_{k,k} - (K\mathbb{N}_k)_{,k}]N_{ij} = 0, \qquad m_{ij} = [S_{ijk} + (Q_k + K\mathbb{N}_k)N_{ij}]n_k \tag{10}$$

According to the notion of Gurtin (2003), Eq. $(9)_1$ expresses the *macroforce balance*, Eq. $(9)_2$ provides the local *macrotraction boundary conditions* on forces, Eq. $(10)_1$ is the *microforce balance* detailed in the next subsection, and Eq. $(10)_2$ is the *microtraction condition*, which is a higher-order internal boundary condition augmented by the interaction of dislocations across interfaces. The microtraction condition, Eq. $(10)_2$, is the soul of this paper as presented in Section 2.3.

By applying the nonlocal Clausius–Duhem inequality as derived by Abu Al-Rub et al. (2007), one can derive the following local and nonlocal conjugate forces:

$$\sigma_{ij} = \rho\frac{\partial\Psi^{\mathrm{e}}}{\partial\varepsilon^e_{ij}}, \qquad X_{ij} = \rho\frac{\partial\Psi^{\mathrm{p}}}{\partial\varepsilon^p_{ij}}, \qquad R = \sigma_y + \rho\frac{\partial\Psi^{\mathrm{p}}}{\partial p}, \qquad S_{ijk} = \rho\frac{\partial\Psi^{\mathrm{p}}}{\partial\varepsilon^p_{ij,k}}, \qquad Q_k = \rho\frac{\partial\Psi^{\mathrm{p}}}{\partial p_{,k}}, \qquad K = \rho\frac{\partial\Psi^{\mathrm{p}}}{\partial e} \tag{11}$$

where ρ is the material density and Ψ^{e} and Ψ^{p} are the elastic and plastic Helmholtz free energy densities, respectively.

2.2. Nonlocal plasticity yield criterion

One can view the *microforce balance* in Eq. $(10)_1$ as the plasticity nonlocal yield condition. By taking the Euclidean norm $\|\|$ of Eq. $(10)_1$, the nonlocal plasticity loading surface f can then be expressed as

$$f = \|\tau_{ij} - X_{ij} + S_{ijk,k}\| - R + Q_{k,k} + (K\mathbb{N}_k)_{,k} = 0 \tag{12}$$

where N_{ij} is collinear with $\tau_{ij} - X_{ij} + S_{ijk,k}$. It is obvious that Eq. (12) represents a sphere in deviatoric stress-space of radius $R - Q_{k,k} - (K\mathbb{N}_k)_{,k}$ centered at $X_{ij} - S_{ijk,k}$. One can also notice that the higher-order stress $S_{ijk,k}$ is a backstress quantity giving rise to additional kinematic hardening (i.e. Bauschinger size-dependent effect), while the microstresses $Q_{k,k}$ and $(K\mathbb{N}_k)_{,k}$ are giving rise to additional isotropic hardening (i.e. strengthening).

In order to develop equations amenable to the analysis and computation, one now considers a simple example for the definition of the Helmholtz free energy function. Both Ψ^{e} and Ψ^{p} that appear in Eq. (11) can be assumed to have, respectively, the following quadratic form:

$$\rho\Psi^e = \frac{1}{2}\varepsilon^e_{ij}E_{ijkl}\varepsilon^e_{kl}, \qquad \rho\Psi^p = \frac{1}{2}hE^2 \tag{13}$$

where E_{ijkl} is the symmetric fourth-order elastic stiffness tensor and h is the hardening modulus. The parameter $E = \int_0^t \dot{E} dt$ is the generalized total accumulation of plastic strain and plastic strain gradients that is intended to measure the total dislocation density (statistically stored and geometrically necessary dislocations), which is defined in Eq. (4).

Making use of Eq. (13) into Eq. (11) the following laws are obtained:

$$\sigma_{ij} = E_{ijkl}(\varepsilon_{kl} - \varepsilon^p_{kl}), \qquad X_{ij} = h\varepsilon^p_{ij}, \qquad R = \sigma_y + hp \tag{14}$$

$$S_{ijk} = h\ell^2 \varepsilon^p_{ij,k}, \qquad Q_k = h\ell^2 p_{,k}, \qquad K = h\ell^2 e \tag{15}$$

Substituting the above equations into the yield function f, Eq. (12), one can then write

$$f = \|\tau_{ij} - h\varepsilon^p_{ij} + h\ell^2 \nabla^2 \varepsilon^p_{ij}\| - \sigma_y - h[p - \ell^2(\nabla^2 p + \|\nabla_i p\| \mathbb{N}_{k,k} + e_{,k}\mathbb{N}_k)] = 0 \tag{16}$$

where ∇^2 designates the Laplacian operator. In the absence of plastic strain gradients, the classical von-Mises criterion is retrieved.

2.3. Interfacial effects

Plastic deformation is mainly carried by dislocations within the bulk (e.g. individual grains). Dislocations can move through the crystal grains and can interact with each other. Interfaces (e.g. Grain boundaries) often hinder their transmission, creating a dislocation pile-up at the interface and thereby making the material harder to deform. Therefore, plastic deformation is strongly affected by interfaces. Following the framework of Gudmundson (2004), Aifantis and Willis (2005), Abu Al-Rub et al. (2007), it will be shown here that the microscopic boundary condition in Eq. $(10)_1$ is related to the interfacial energy at free surfaces (e.g. the surface of a freestanding thin film, the free surface of a void) or interfaces (e.g. the film-substrate interface, grain boundaries, inclusion interface). Interfacial energy in small-scale systems (e.g. thin films, nano-wires) is significant and cannot be ignored when the surface-to-volume ratio becomes large enough. In Eq. $(10)_2$, the micro-traction stress m_{ij} is meant to be the driving force at the material internal and external boundaries, which can be interpreted as the *interfacial stress* at free surface or interface which is conjugate to the surface plastic strain. Therefore, m_{ij} can be related to the interfacial energy φ per unit surface area by using the well-known relation:

$$m_{ij} = \frac{\partial \varphi(\varepsilon^{p(I)}_{ij})}{\partial \varepsilon^{p(I)}_{ij}} \quad \text{on} \quad \partial\Gamma^p \tag{17}$$

where $\varepsilon^{p(I)}_{ij}$ is the surface plastic strain and $\partial\Gamma^p$ is the plastic surface. Hence, constrained plastic flow could be modeled either as a full constraint, i.e. $\varepsilon^{p(I)}_{ij} = 0$ (when $\varphi \to \infty$), or no constraint, i.e. $m_{ij} = 0$ (when $\varphi \to 0$). An intermediate kind of micro-boundary condition can be described by defining a definite value for the interfacial energy at the interface such that some dislocations are allowed to transfer across the interface and some are pilled-up at the interface. Abu Al-Rub (in press) has investigated different expressions for the interfacial energy and found out that the following expression gives close qualitative agreement with experimentally observed size effect, such that:

$$\varphi = \frac{1}{2}\ell_I(\sigma_y \|\varepsilon^{p(I)}_{ij}\| + h\varepsilon^{p(I)}_{ij}\varepsilon^{p(I)}_{ij}) \quad \text{on} \quad \partial\Gamma^p \tag{18}$$

where σ_y is the bulk (size-independent) yield strength, h is the strain hardening modulus, and ℓ_I is another microstructural length scale that is related to boundary layer thickness and characterizes the stiffness of the interface boundary in resisting plastic deformation. If $\ell_I = 0$, the interface would behave like a free surface and one obtains a micro-free boundary condition (i.e. $m_{ij} = 0$). On the other hand, if $\ell_I \to \infty$ then it would represent a condition for fully constrained dislocation movement at the interface and one obtains a micro-clamped boundary condition (i.e. $\varepsilon^{p(I)}_{ij} = 0$).

The microtraction stress at the boundary, $\mathbf{m}$, can then be obtained from Eqs. (17) and (18) as

$$m_{ij} = \ell_I(\sigma_y \varepsilon^{p(I)}_{ij} / \|\varepsilon^{p(I)}_{mn}\| + h\varepsilon^{p(I)}_{ij}) \quad \text{on} \quad \partial\Gamma^p \tag{19}$$

such that for $\varepsilon_{ij}^{p(I)} = 0$, $m_{ij} = \pm \ell_I \sigma_y \delta_{ij}$ such that $\ell_I \sigma_y$ characterizes the interfacial yield strength. Therefore, Eq. (19) physically characterizes a bulk-like yield condition at the interface, which governs the plasticity at the interface.

3. Application to size effects in thin films

This section presents an application of the proposed gradient plasticity model to handle size effects in metallic thin films. This model is used to investigate the size-dependent behavior in uniaxial loading of a plastic thin film on an elastic substrate [see Fig. 1(a)]. The nonlocal yield function in Eq. (16) is solved numerically using the finite element algorithm for gradient plasticity as detailed in Abu Al-Rub and Voyiadjis (2005). Readers are referred to this paper for more details.

Results in Figs. 1(b) and (c), 2(a) and (b) are presented for $h/E = 0.2$ for film thicknesses as represented by $\ell/d = 0.1$, 0.5, 1, 1.5, and 2. The level of interfacial energy at the interface is controlled by the normalized ratio ℓ_I/ℓ. Fig. 1(b) and (c) show, respectively, the non-uniform plastic strain distribution across the film thickness d and the average stress–strain relation for $\ell_I/\ell = 1.0$. It is obvious from Fig. 1(c) that both the overall (i.e. macroscopic) yield strength of the film and the strain hardening rate increase with decreasing the characteristic size d. This response conforms to the experimental results at the micron and submicron length scales (see for example Huang and Spaepen, 2000; Haque and Saif, 2003; Espinosa et al., 2004). This result is confirmed in Fig. 2(a) and (b) by plotting the normalized overall yield strength, σ_Y/σ_y, and the normalized strain hardening rate, E^{T}/E, as a function of the normalized film thickness for different interfacial strengths as characterized by the ratio ℓ_I/ℓ. It can be seen that a maximum increase in the yield strength and the strain hardening rate is obtained by assuming a rigid interface where dislocations are not allowed to transmit across the interface but instead piles-up there. Softer responses are obtained by reducing the interfacial strength. This indicates that for a rigid interface, d alone (represented by the ratio ℓ/d) controls the increase in the yield strength, whereas for compliant and intermediate interfaces both d and ℓ_I determine the yield strength and strain hardening rate.

Therefore, from Fig. 2(a) and (b), one can conclude that if the film thickness is kept constant while the interfacial strength is increased, an increase in both the macroscopic yield strength and strain hardening rates

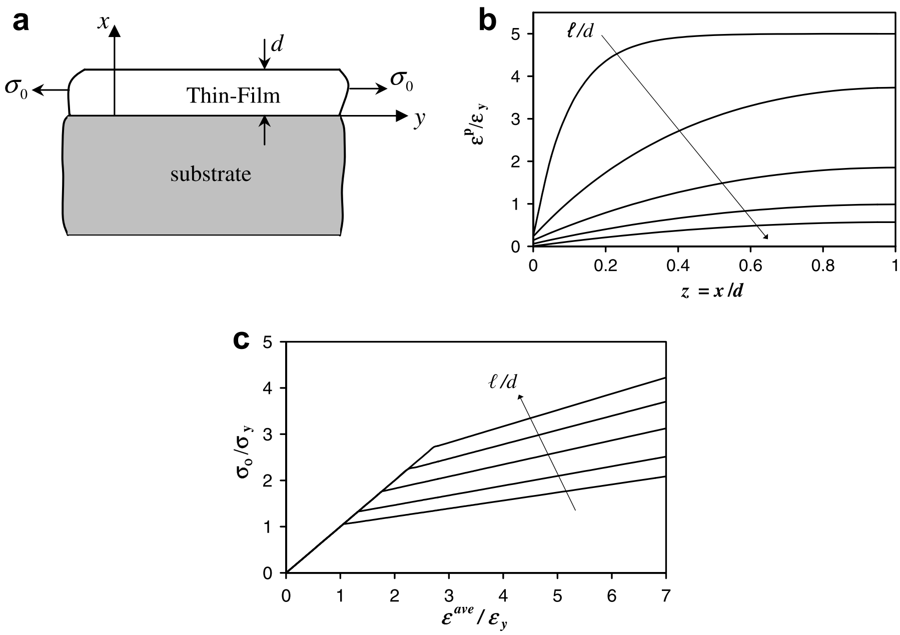

Fig. 1. (a) thin film on an elastic substrate subjected to uniaxial tension. (b) Normalized plastic strain distribution along d for $\sigma_0/\sigma_y = 2$ and $\ell_I/\ell = 1.0$. (c) Normalized stress–strain relations for $\ell_I/\ell = 1.0$. Different sizes are represented by $\ell/d = 0.1$, 0.5, 1, 1.5, 2.

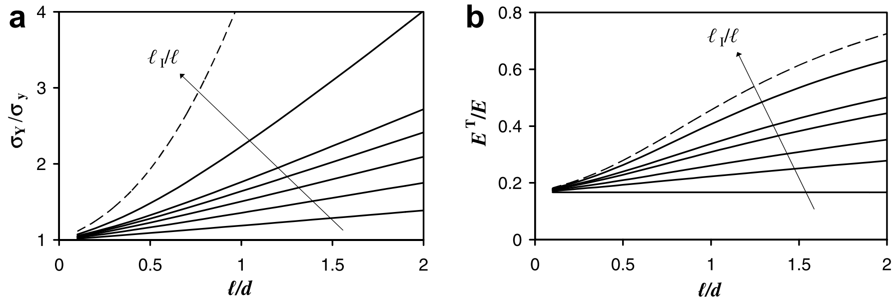

Fig. 2. Normalized (a) yield strength and (b) tangent modulus vs. ℓ/d for $\ell_I/\ell = 0.2, 0.4, 0.6, 0.8, 1, 2$, and ∞. The dashed line is for a rigid interface (i.e. $\ell_I/\ell \to \infty$).

is observed until a critical value of the interfacial strength is reached that corresponds to a rigid interface where dislocations are completely blocked such that a further increase in the interfacial strength will not influence anymore the observed size effect. On the other hand, if a rigid interface is assumed, then the reduction in the film thickness (or equivalently an increase in the bulk length scale ℓ) will increase both the macroscopic yield strength and strain hardening rates. If an intermediate interface is assumed, then both the film thickness (or ℓ) and the interfacial strength (or ℓ_I) contribute to the observed size effect.

4. Conclusions

The most crucial conclusion of the present study is that if the interface is weak or compliant in terms of constraining plastic deformation, it determines the yield strength and strain hardening of the specimen. In this case, the interfacial length scale, ℓ_I, will control the size effects. On the other hand, if the interface is stiff or rigid in terms of constraining plastic deformation (i.e. having an interface with a very high interfacial strength), a further increase in the interfacial strength (or equivalently increasing the interfacial length scale ℓ_I) will not influence the yield strength and strain hardening of the specimen anymore. In this case, the bulk length scale, ℓ, or the reduction in the film thickness will control the size effect. This means that size effect is controlled by the weakest link of bulk and interface.

The formulation of higher-order boundary conditions is very important within strain gradient plasticity theory, especially, at interfaces, grain, or phase boundaries. It is shown that interfacial effects can be considered by relating the microtractions at interfaces to the interfacial energy which is dependent on the plastic strain at the interface. This is an important aspect for further development of gradient-dependent plasticity that is capable of modeling size effects in micro/nano-systems. It is shown that the existence of both gradients and interfacial energies contribute to the observed size effects.

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