1 Introduction

Advanced metal matrix composites (AMMCs) are defined as pure metals or metallic alloys with dispersed particles or inclusions at decreasing microstructural length scales ranging in size from a few micrometers down to hundreds of nanometers. These AMMCs exhibit mechanical and physical properties that are improved over those exhibited by conventional metal matrix composites (i.e., pure metals or metallic alloys that consist of large particles, fibers, whiskers, etc.). They also have considerable economic, technological, and scientific interest and will contribute significantly to the heightened performance of structural components and systems in many industries [1]. For example, Fig. 1 shows the microstructure of a structural steel embedded with elliptical inclusions at two distinct scales: primary particles are typically on the order of several microns in size, and secondary particles that range from a few micrometers down to hundreds of nanometers [2].

The strengthening mechanism of AMMCs generally includes two types: direct and indirect strengthening. Direct strengthening results from load transfer from the metal matrix to the reinforcing particle. Indirect strengthening results from the matrix microstructure (e.g., finer grain size) or heterogeneous plastic deformation due to a plastic strain incompatibility at the matrix-particle interface. Plastic strain incompatibility is accommodated by the formation of geometrically necessary dislocations (GNDs) that scale with plastic strain gradients [3]. Experiments have shown that the macroscopic behavior of metallic alloys and composites depends not only on the volume fraction of dispersed particles within the metallic matrix but also on their size, shape, and distribution. Unfortunately, classical (local) continuum plasticity theories are unable to predict the indirect strengthening such as the influence of GNDs at the particle-matrix interface due to the absence of a material length scale in their constitutive equations. However, the mechanical properties of AMMCs, with a typical distance $D$ between particles and particle size $d$, depend on how these geometrical parameters interfere with other characteristic distances such as the mean-free path of dislocations $l$. It was shown that the increase in strength with decreasing scale can be related to a proportional increase in the plastic strain gradients where the gradient plasticity theory has been successful in addressing the size effect problem (see Refs. [1] and [4–8] for a comprehensive review and the reference quoted therein). This success stems from the incorporation of a microstructural length scale parameter through functional dependencies on the plastic strain gradient of nonlocal media. The gradient-dependent theory abandons the assumption that the stress at a given point is uniquely determined by the strain at this point only. This theory takes into account possible interactions with other material points in the vicinity of the point of interest (see Ref. [5] for a comprehensive review of lower- and higher-order gradient plasticity theories and the reference quoted therein).

The objective of this paper is to extend and to utilize the preceding frameworks of gradient plasticity to address the problem of strain hardening in AMMCs due to size and orientation of particles or inclusions. A higher-order gradient plasticity theory is formulated using the principle of virtual power and the laws of thermodynamics utilizing the general thermodynamic framework of Abu Al-Rub et al. [5]. The proposed gradient plasticity theory is applied
for investigating the behavior of a unit cell of a metallic matrix embedded with elliptic inclusions of different sizes, aspect ratios, and orientation, while keeping the volume fraction of particles constant. Special attention will be given to the overall average stress–strain response computed from a unit cell and the distributions of von Mises, plastic strain, and plastic strain gradients.

2 Thermodynamics of Gradient Plasticity Theory

In order to be able to model the small-scale phenomena, such as the effect of size of microstructural features (intrinsic size effect) and the effect of geometric size (extrinsic size effect) on the material mechanical properties, an attempt is made now to account for the effect of plastic strain gradients. This is done by developing a higher-order gradient-dependent plasticity theory using the principle of virtual power and the laws of thermodynamics. The following theory is based on the general thermodynamic framework presented in Ref. [5].

Based on the crystallographic dislocation basis presented in Ref. [5], it is shown that both the gradients of the plastic strain tensor, ∇εp, and the effective plastic strain, ∇p, should enter the definition of the internal virtual work besides their corresponding time derivative, using the current time, such that one cannot exist without the other. Therefore, the principle of virtual power, which is the assertion that the virtual power done by external forces, such that the rate of change in the total free energy should be less than or equal to the external power, into Eq.(6), and then localize for the virtual power expended within Ω (i.e., internal power), can be expressed as follows:

\[
\int_{\Omega} \left( \sigma_{ij} \delta v^i_j + X_{ij} \delta v^i_j + R \delta \dot{p} + S_{ik} \delta \dot{v}^i_{jk} + Q_{ik} \delta \dot{v}^j_k \right) d\Omega = \int_{\partial \Omega} \left( t^i \delta v^i_j + m_{ij} \delta \dot{v}^i_j \right) d\partial \Omega
\]

where \( \sigma \) is the Cauchy stress tensor, \( \varepsilon^p = \varepsilon - \varepsilon^d \) (assuming small deformations) is the elastic strain tensor with \( \varepsilon \) being the total strain tensor and \( \varepsilon^d \) is the plastic strain tensor, \( X \) is the backstress tensor associated with local kinematic hardening, \( R \) is the dragstress associated with local isotropic hardening, \( S \) is a higher-order stress tensor associated with nonlocal kinematic hardening, \( Q \) is a higher-order force vector associated with nonlocal isotropic hardening, \( t \) is the macroscopic surface traction vector, \( \gamma \) is the velocity vector, \( m \) is a higher-order moment stress tensor that is prescribed at the surface or interface boundary \( \partial \Omega \) with an outward unit normal \( n \). Note that \( \delta \) is the standard variation parameter and is used here to designate a virtual quantity, the superimposed dot designates the derivative with respect to time, and the comma is used to designate a derivative with respect to \( x_i \).

Substituting \( \varepsilon^d = \dot{\varepsilon} - \dot{\varepsilon}^d \) assuming incompressible plastic flow (i.e., \( \delta_{ik} = 0 \)), \( \varepsilon = \nabla \cdot \varepsilon \), and \( \varepsilon^d = pN \), with \( N \) being the direction of the plastic flow (i.e., the plasticity flow rule) where its magnitude is \( \|N\| = \sqrt{3}/2 \), into the virtual power balance, Eq. (1), and then applying the divergence theorem yields, after some lengthy manipulations, the following result:

\[
\int_{\Omega} \sigma_{ij} \delta v^i_j d\Omega + \int_{\partial \Omega} \left( t^i - \sigma_{ij} n_j \right) \delta v^i_j d\partial \Omega \\
+ \int_{\Omega} \left[ \tau_j - X_{ij} + S_{ik,k} - \left( R - Q_{ik,k} \right) N_j \right] \delta \dot{v}^i_j d\Omega \\
+ \int_{\partial \Omega} \left[ n_j - \left( S_{ik,k} + Q_{ik} N_j \right) n_l \right] \delta \dot{v}^i_j d\partial \Omega = 0
\]

(2)

where \( \tau \) is the deviatoric part of \( \sigma \) (i.e., \( \tau_j = \sigma_{ij} - 1/3 \sigma_{kk} \delta_{ij} \) where \( \delta_{ij} \) is the Kronecker delta). The fields \( \Gamma, \delta v, \) and \( \delta \dot{v} \) in Eq. (2) may be arbitrarily specified if and only if

\[
\sigma_{ij} = 0 \quad (3a) \\
t^i = \sigma_{ij} n_j \quad (3b) \\
\tau_j - X_{ij} + S_{ik,k} - \left( R - Q_{ik,k} \right) N_j = 0 \quad (4a) \\
m_{ij} = \left( S_{ik,k} + Q_{ik} N_j \right) n_l \quad (4b)
\]

Equation (3a) expresses the macroforce balance; Eq. (3b) defines the stress vector as the surface density of external forces and provides the local macrotraction boundary condition on forces; Eq. (4a) is the microforce balance; and Eq. (4b) is the microtraction condition, which is a higher-order boundary condition augmented by the interaction of dislocations across internal interfaces or external surfaces [5].

By taking the Euclidian norm of Eq. (4a) and assuming that the plastic flow direction is codirectional to the stress tensor, one can show that the microforce balance is the nonlocal form of the von Mises plasticity yield function, such that

\[
\left\| \frac{3}{2} \tau_j - X_{ij} + S_{ik,k} \right\| - R + Q_{ik,k} = 0
\]

(5)

The constitutive equations for the thermodynamic forces \( \sigma, X, S, R, \) and \( Q \) can be obtained by making use of the nonlocal Clausius–Duhem inequality for isothermal conditions [1,5], which can be derived by using the first- and second-laws of thermodynamics that reduce under isothermal conditions to the statement that the rate of change in the total free energy should be less than or equal to the power done by external forces, such that

\[
\int_{\Omega} \rho \dot{\Psi} d\Omega \leq \int_{\partial \Omega} \left( t^i \delta v^i_j + m_{ij} \delta \dot{v}^i_j \right) d\partial \Omega
\]

(6)

where \( \rho \) is the material density, and \( \Psi = \Psi(\varepsilon, \varepsilon^d, \rho, \nabla \varepsilon, \nabla \rho) \) is the Helmholtz free energy per unit mass, which is a function of the local and gradient-dependent internal state variables. Substituting the actual (not virtual) internal power from Eq. (1), which is equal to the external power, into Eq. (6), and then localize for any arbitrary subvolume \( \Omega \), the following nonlocal Clausius–Duhem inequality can then be obtained:

\[
\Pi = \sigma_{ij} \delta_{ij} - \rho \dot{\Psi} + \Sigma \geq 0
\]

(7)

where \( \Pi \) is the rate of plastic energy dissipation, and \( \Sigma \) designates the nonlocal energy residual that results from nonlocal long-range microstructural interactions between the material points in the active plastic zone and interfaces, and is given by

\[
\Sigma = \left( S_{ik,k} + Q_{ik} \right) N_j
\]

(8)

which shows that in the absence of plastic strain gradients, one retains the local Clausius–Duhem inequality such that \( \Sigma = 0 \).
Substituting the time derivative of $\Psi$ into Eq. (7) along with Eq. (8) and $\dot{e} = \dot{e}^c + \dot{e}^p$ yields the following expression:

$$
\Pi = \left( \sigma_{ij} - \frac{\partial \Psi}{\partial \epsilon_{ij}^c} \right) \dot{e}_{ij}^c + \left( \dot{X}_{ij} - \rho \frac{\partial \Psi}{\partial \dot{p}} \right) \dot{p}_{ij} + \left( R - \rho \frac{\partial \Psi}{\partial \dot{p}} \right) \dot{p} + \left( S_{ijk} - \rho \frac{\partial \Psi}{\partial \dot{p} \dot{p}_k} \right) \dot{p}_{ij} \dot{p}_k + \left( Q_k - \rho \frac{\partial \Psi}{\partial \dot{p} p_k} \right) \dot{p}_k \geq 0
$$

(9)

In order not to end up with zero $\Pi$, one needs to define the thermodynamic conjugate forces that are related to the free energy as energetic forces, such that one can define

$$
\sigma_{ij} = \rho \frac{\partial \Psi}{\partial \epsilon_{ij}^c}, \quad X_{ij} = \rho \frac{\partial \Psi}{\partial \epsilon_{ij}^c}, \quad R = \rho \frac{\partial \Psi}{\partial \dot{p}}, \quad S_{ijk} = \rho \frac{\partial \Psi}{\partial \dot{p} \dot{p}_k}, \quad Q_k = \rho \frac{\partial \Psi}{\partial \dot{p} p_k}
$$

(10)

where $X^c, R, S^0,$ and $Q^0$ are the energetic components of $X, R, S,$ and $Q$, respectively. Therefore, one can rewrite Eq. (9) as

$$
\Pi = \left( X_{ij} - X_{ij}^c \right) \dot{e}_{ij}^c + \left( R - R^c \right) \dot{p} + \left( S_{ijk} - S_{ijk}^0 \right) \dot{p}_{ij} \dot{p}_k + \left( Q_k - Q_k^0 \right) \dot{p}_k \geq 0
$$

(11)

where $X^0, R^0, S^0,$ and $Q^0$ are the dissipative components of $X, R, S,$ and $Q$, respectively. This implies that each conjugate force associated with the plasticity internal state variables should be additively decomposed into an energetic component and a dissipative component, such that

$$
X_{ij} = X_{ij}^c + X_{ij}^0, \quad R = R^c + R^0, \quad S_{ijk} = S_{ijk}^0 + S_{ijk}^c, \quad Q_k = Q_k^0 + Q_k^c
$$

(12)

Now, the question is: how one can derive an expression for the dissipative conjugate forces? In fact, this can be achieved through using the maximum rate of dissipation principle that states that the material dissipates energy in the easiest way possible. This maximization can be achieved through using the calculus of Lagrange multipliers by using the auxiliary function

$$
\Omega = \Pi - \lambda D
$$

(13)

where $\lambda$ is the Lagrange multiplier and $D = 0$ is the constraint for a non-negative rate of energy dissipation, which is defined from Eq. (11) as

$$
D = \Pi - X_{ij}^0 \dot{e}_{ij}^c + R^0 \dot{p} + S_{ijk}^0 \dot{p}_{ij} \dot{p}_k + Q_k^0 \dot{p}_k = 0
$$

(14)

The necessary conditions to maximize $\Omega$ are as follows:

$$
\frac{\partial \Omega}{\partial \dot{e}_{ij}^c} = 0, \quad \frac{\partial \Omega}{\partial \dot{p}} = 0, \quad \frac{\partial \Omega}{\partial \dot{p}_k} = 0
$$

(15)

Substituting Eqs. (13) and (14) into Eq. (15) yields

$$
\dot{X}_{ij}^c = \lambda \frac{\partial \Omega}{\partial \epsilon_{ij}^c}, \quad \dot{R}^0 = \lambda \frac{\partial \Omega}{\partial \dot{p}}, \quad \dot{S}_{ijk}^0 = \lambda \frac{\partial \Omega}{\partial \dot{p}_k}, \quad \dot{Q}_k^0 = \lambda \frac{\partial \Omega}{\partial \dot{p}_k}
$$

(16)

where $\lambda = (\Lambda - 1)/\Lambda$, which can be determined by substituting Eq. (16) into Eq. (11) as

$$
\lambda = \frac{\partial \Pi}{\partial \dot{e}_{ij}^c} \dot{e}_{ij}^c + \frac{\partial \Pi}{\partial \dot{p}} \dot{p} + \frac{\partial \Pi}{\partial \dot{p}_k} \dot{p}_k
$$

(17)

Therefore, in deriving constitutive relations for the energetic and dissipative thermodynamic forces, one needs to know: (1) how the material stores energy by assuming an expression for the Helmholtz free energy function, $\Psi$; and (2) how the material dissipates energy by assuming an expression for the rate of energy dissipation function, $\Pi$. Therefore, the above thermodynamic framework is general and can be used to derive any local/nonlocal constitutive models through only assuming expressions for $\Psi$ and $\Pi$.

For simplicity, the following forms for $\Psi$ and $\Pi$ are assumed here in order to derive rate-independent gradient-dependent plasticity theory, such that

$$
\rho \Psi = G \epsilon_{ij}^c \epsilon_{ij}^c + \frac{1}{2} \frac{\dot{h}}{h} \dot{h} + \frac{1}{2} G \dot{e}_{ij}^c \dot{e}_{ij}^c
$$

(18)

$$
\Pi = 2 \sigma \dot{p} + 2 G \dot{e}_{ij}^c \dot{e}_{ij}^c
$$

(19)

where $G = E/(1 + \nu)$ is the shear modulus, $\lambda = 2\nu G/(1 - 2\nu)$ is the Lamé’s constant with $E$ being the Young’s modulus and $\nu$ is the Poisson’s ratio, $h$ is the plasticity hardening modulus, $\sigma$ is the size-independent initial yield strength, $s > 0$ is the first-order gradient exponent, and $\ell$ is the material length scale, which is assumed constant in this study (see Ref. [9] for a discussion on the physical and experimental interpretation of $\ell$).

Substituting Eqs. (18) and (19) into Eqs. (10) and (16) along with Eq. (12) yields

$$
\sigma_{ij} = 2 G \epsilon_{ij}^c + \lambda \dot{h} \delta_{ij}
$$

(20)

$$
X_{ij} = h \epsilon_{ij}^c + \frac{2}{3} \frac{\sigma_y + G \dot{e}_{ij}^c \dot{e}_{ij}^c}{\dot{p} \dot{p}_k} N_{ij},
$$

(21)

$$
R = \sigma_y + \dot{h} \dot{p} + G \dot{e}_{ij}^c \dot{e}_{ij}^c
$$

$$
S_{ijk} = G \dot{e}_{ij}^c \dot{e}_{ij}^c, \quad Q_k = G \dot{e}_{ij}^c \dot{e}_{ij}^c
$$

(22)

where $\Lambda = 1/2$ is obtained from Eq. (17). Then, one can express the nonlocal yield condition in Eq. (5) as follows:

$$
\frac{f}{\eta} \left[ \frac{3}{2} \tau_{ij} - h \dot{e}_{ij}^c - \frac{2}{3} \frac{(\sigma_y + G \dot{e}_{ij}^c \dot{e}_{ij}^c)}{\dot{p} \dot{p}_k} N_{ij} = h \ell^2 \nabla^2 \epsilon_{ij}^c \right]
$$

$$
- \sigma_y - \dot{h} \dot{p} + G \dot{e}_{ij}^c \dot{e}_{ij}^c + G \dot{e}_{ij}^c \dot{e}_{ij}^c = 0
$$

(23)

where $\nabla^2$ designates the Laplacian operator. In the absence of plastic strain gradients, the classical von Mises criterion is retrieved.

For more meaningful presentation of the effect of plastic strain gradients through the unit cell simulations in Sec. 4, the density of geometrically necessary dislocations, $\rho_G$, will be computed using the following relation [10]:

$$
\rho_G = \frac{\| p_k \|}{b}
$$

(24)

where $b$ is the magnitude of the Burgers vector.

The formulation presented in this section has been implemented through the user material subroutine User Material (UMAT) in the well-known commercial finite element software ABASYS. A direct numerical implementation algorithm for gradient-dependent plasticity in the sense that the nonlocal consistency condition, Eq. (23), is transformed into a linear set of equations that depends on the material parameters, and the current co-ordinates of the integration points have been adapted following the work in Ref. [11].
Fig. 2 The plane stress unit cell model for an elliptical particle. (a) Assumed periodically arranged reinforcement in the overall inhomogeneous material. (b) The unit cell used for modeling is shown with the finite element mesh.

Fig. 3 Simulation results for $L_0 = 10 \mu m$ and $a_0/b_0 = 1$ showing contours of von Mises stress (in MPa) in (a) and (b); effective plastic strain in (c) and (d); and density of GNDs (in 1/m$^2$) in (e) at 5% applied strain level. Here (a) and (c) show results from classical plasticity theory (i.e., for $\ell = 0$), while (b), (d), and (e) show results from the current gradient plasticity theory.
3 Finite Element Model

A composite material having a periodical distribution of inclusions is analyzed using a plane stress unit cell approach. Thus, the results presented in this study approximate a composite of rather long, almost aligned, stiff inclusions, which are subjected to a monotonic tensile stress state in the vertical direction (i.e., transverse to the orientation of the inclusion) of the unit cell (see Fig. 2). Figure 2(a) shows the overall inhomogeneous material with periodically distributed elliptical inclusions, and Fig. 2(b) shows the corresponding unit cell discretized with a fine finite element mesh. The cross-section of the inclusion is assumed elliptical and defined by the geometric features, $a_0$ and $b_0$, while the unit cell is square with the dimension of $L_0$ that represents the interparticle spacing (see Fig. 2). The inclusion’s aspect ratio, $a_0/b_0$, can be used to model the morphology of the inclusion; therefore, three different ratios (1, 3, and 6) were simulated such that $a_0/b_0 = 1$ represents a circular inclusion. Tensile displacement control loading was applied on the upper surface of the unit cell, while imposing periodic boundary conditions at the right and left surfaces. For the following simulations, the volume fraction of the inclusion in the unit cell, given by $\alpha a_0 b_0 / 4L_0^2$, is kept constant as 5%.

Fig. 4 Overall average stress–strain responses predicted by classical ($L_0/\ell \to \infty$) and gradient plasticity models for different interparticle spacing $L_0 = 40, 20, 10\mu m$ and different inclusion’s aspect ratio at the same particle volume fraction.

Fig. 5 Simulation results for $L_0 = 10\mu m$ and $a_0/b_0 = 3$ showing contours of von Mises stress (in MPa) in (a) and (b); effective plastic strain in (c) and (d); and density of GNDs (in 1/m$^2$) in (e) at 5% applied strain level. Here (a) and (c) show results from classical plasticity theory (i.e., $\ell = 0$), while (b), (d), and (e) show results from the current gradient plasticity theory.
In this paper, the behaviors of the inclusion and the matrix are assumed elastic and linear elastic–plastic, respectively. The material constants for the matrix are assumed to be $E = 10^2$ GPa, $
u = 0.3$, $\sigma_y = 160$ MPa, $h = 7$ GPa, $s = 1$, $\ell = 0.3 \mu m$ and $b = 0.25$ nm, whereas the inclusion’s elastic material constants are assumed as $E = 420$ GPa and $\nu = 0.17$.

4 Numerical Results

In this section, the results from different unit cell simulations, as the one in Fig. 2(b) under vertical uniaxial tensile displacement (see Fig. 2(b)), are presented. In order to investigate the effect of the inclusion interparticle spacing on the overall average mechanical behavior, unit cell simulations were conducted for different sizes of the unit cell, $L_0$, while the volume fraction of inclusions is kept constant at 5%. Figures 3(a)–3(d) compare contours of effective von Mises stress and effective plastic strain for classical plasticity and gradient enhanced plasticity model for a unit cell of size of $L_0 = 10 \mu m$ and an aspect ratio of $a_0/b_0 = 1$ at 5% applied strain level (classical plasticity corresponds to the limit $L_0/\ell \to \infty$). As one can see in Figs. 3(a) and 3(b), the effective von Mises stress distribution is significantly higher once plastic strain gradients are considered; notably more localized around the inclusion where the gradient of effective plastic strain has much more impact on the material behavior. A comparison between these contours reveals that, unlike classical plasticity model (Fig. 3(a)) where strengthening results from the load transfer, from the metal matrix to the stiff reinforcing inclusion, strengthening results from changes in the deformation mode in the matrix material near the inclusion-matrix interface due to the high strain gradient effect. In Figs. 3(c) and 3(d), one can also see that the plastic strain gradient effect changes the whole plastic strain distribution as compared to the results from classical plasticity such that the value of plastic strain reduces drastically near the inclusion interface. Figure 3(e) shows the density of GNDs, which contributes to additional isotropic hardening in the nonlocal yield function, Eq. (18), due to evolution of plastic strain gradients. As expected, the maximum value of GNDs occurs around the inclusion. Therefore, all changes seen in the von Mises stress and effective plastic strain distributions, when applying the gradient enhanced model as compared to that from classical plasticity, originate from considering gradient-hardening effects.

Similarly, Figs. 5 and 6 show the contours of effective von Mises stress, effective plastic strain, and density of GNDs for the other two different inclusion’s aspect ratios $a_0/b_0 = 3$ and $a_0/b_0 = 6$, respectively, for a unit cell of size $L_0 = 10 \mu m$. The corresponding average stress–strain responses are shown in Fig. 4. Interestingly, unlike the results from the classical plasticity theory
(i.e., Figs. 3(c), 5(c), and 6(c)), one can see in Figs. 3(d), 5(d), and 6(d) that the effective plastic strain is almost zero around the inclusion indicating that the stiff inclusion enforces zero plastic strain around the inclusion-matrix interface, which is attributed to the high level of additional gradient hardening (see Figs. 3(e), 5(e), and 6(e)). Accordingly, one would expect that enforcing higher-order boundary conditions at the inclusion-matrix interface, Eq. (4b), through imposing a hard boundary condition (i.e., zero plastic strain at the interface) as done in Refs. [1] and [4] will not have a significant effect on the plastic strain distribution and the stress–strain response. Therefore, these higher-order boundary conditions, Eqs. (4b), have not been enforced in the current simulations. Moreover, unlike that for the circular inclusion (see Fig. 3(e)), one can notice that the distribution of GNDs density is more localized around the inclusion tip in the case of an elliptical inclusion (i.e., a0/b0 = 3 or 6) such that the average GNDs density is higher in the case of circular inclusion. Hence, as shown in Fig. 4, the stress–strain response of the unit cell of L0 = 10 μm with a circular inclusion is higher.

Figure 4 shows the overall average stress–strain responses predicted by classical and gradient-enhanced plasticity for different sizes of the unit cell L0 (i.e., different interparticle spacing) and different a0/b0 ratios. By using classical plasticity (i.e., L0/ε → ∞), no considerable effect is seen for different inclusion’s aspect ratios (i.e., no shape effect) and different interparticle spacing (i.e., no size effect). On the other hand, it is interesting to note that when considering the gradient-enhanced plasticity, the average stress–strain response is more sensitive to the inclusion’s aspect ratio as L0 decreases (or equivalently as √a0b0 decreases). This is attributed to the higher GNDs density as L0 or √a0b0 decreases. Moreover, for L0/ε → ∞, L0 = 20 μm, and L0 = 40 μm, the average stress–strain response (particularly the flow stress and strain hardening rate) is stronger for a0/b0 = 6, and almost the same for a0/b0 = 1 and 3. This might be anticipated since the free-path interparticle spacing (i.e., d0 shown in Fig. 2(a)) or equivalently the ratio d0/L0 (d0/L0 = 1 − 4f(a0/b0)/π with f being the volume fraction) decreases as a0/b0 increases that lead to a higher interaction between inclusions in the direction transverse to the loading direction. However, this is not the case for L0 = 10 μm, where the average stress–strain response is stronger for a0/b0 = 1, followed by a0/b0 = 6, and then by a0/b0 = 3. This can be attributed to the density and distribution of GNDs around the inclusion that is completely dependent on L0 and a0/b0. As L0 and a0/b0 increase, the GNDs are more localized in a small region at the inclusion tip; whereas, as L0 and a0/b0 decrease, the GNDs density is more distributed in a larger area surrounding the inclusion (see Figs. 3(e), 5(e), and 6(e)). Therefore, one can conclude that, when the volume fraction is fixed, there is a threshold for the interparticle spacing such that circular inclusions with smaller interparticle spacing will lead to a stronger composite; whereas, elliptical inclusions with higher interparticle spacing and higher aspect ratio perpendicular to the direction of loading lead to a stronger composite.

5 Conclusions

In this paper, a higher-order gradient plasticity model, which includes the effects of both the variation in the effective (equivalent) plastic strain and plastic strain tensor, is formulated based on the principle of virtual power and the nonlocal Clausius–Duhem inequality. The model is then used to investigate the effects of interparticle (or interinclusion) spacing, particle size, and particle aspect ratio in a metal matrix microreinforced composite through plane stress unit cell simulations, while keeping the particle volume fraction constant. This size effect could not be explained by classical plasticity models due to the lack on an intrinsic material length scale parameter in the constitutive description of classical (local) models. By comparing the simulation results from classical plasticity and gradient plasticity, it is shown that the von Mises stress and effective plastic strain distributions in the metal matrix around the inclusion are significantly altered due to the evolution of plastic strain gradients that scale with the density of geometrically necessary dislocations. The contours of effective plastic strain show that the magnitude is higher for the classical plasticity than for gradient-dependent plasticity. Also, the plastic deformation from the classical plasticity is more confined within a small region near the inclusion. Whereas, due to the high level of plastic strain gradient-hardening near the inclusion-matrix interface, a localized zone of near zero plastic strain is seen around the inclusion when considering gradient plasticity, which indicate that the influence of higher-order microscopic boundary conditions, Eq. (4b), at the particle-matrix interface is minor and can be neglected. However, this observation is not conclusive, since this might be dependent on the hardness of the inclusion and the properties of particle-matrix interface.

Based on the obtained average stress–strain responses that show the effect of interparticle spacing for different particle aspect ratios, it is noticed that strengthening due to reducing the particle size is controlled by the free-path interparticle spacing (i.e., d0 in Fig. 2(a)) and the morphology of the inclusion such that when d0 is small enough, circular inclusions lead to larger strengthening than elliptical inclusions. Therefore, decreasing d0 does not necessarily lead to largest strengthening as one might expect. This is completely dependent on the plastic strain gradient-hardening due to distribution and evolution of geometrically necessary dislocations that depend on the particle size and shape.

Finally, due to a lack of careful experimental data on the interparticle spacing effect in the literature, it is very difficult at this point to validate the predictions of either classical plasticity or gradient plasticity theory.

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