A thermodynamic framework for constitutive modeling of time- and rate-dependent materials. Part II: Numerical aspects and application to asphalt concrete

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Abstract

In this paper, we present within the finite element context the numerical algorithm for the integration of the thermodynamically consistent thermo-viscoelastic, thermo-viscoplastic, thermo-viscodamage, and thermo-healing constitutive equations derived in the first part of this paper. The nonlinear viscoelastic model is implemented using a recursive-iterative algorithm, whereas an extension of the classical rate-independent return mapping algorithm to the rate-dependent problems is used for numerical implementation of the viscoplasticity model. Moreover, the healing natural configuration along with the power transformation equivalence hypothesis, proposed in the first part of the paper, are used for the implementation of the viscodamage and micro-damage healing models. Hence, the thermo-viscoelastic and thermo-viscoplastic models are also implemented in the healing configuration. These numerical algorithms are implemented in the well-known finite element code Abaqus via the user material subroutine UMAT. A systematic procedure for identification of model parameters is presented. The model is then used to simulate the time-, temperature-, and rate-dependent response of asphalt concrete over an extensive set of experimental measurements including creep-recovery, creep, triaxial, constant strain rate, and repeated creep-recovery tests in both tension and compression. Comparisons of the model predictions and the experimental measurements show that the model is capable of predicting the nonlinear behavior of asphalt concrete subjected to different loading conditions.

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1. Introduction

Asphalt concrete is comprised of different constituents such as asphalt binder, aggregate, and air voids and shows highly nonlinear mechanical response (e.g. Perl et al., 1983; Sides et al., 1985; Collop et al., 2003; Masad et al., 2008). Several orders of magnitudes of difference between the stiffness of asphalt binder and aggregates, strain localization in the binder phase, rotation and slippage of aggregates, and interaction of binder and aggregates during the loading are the main reasons triggering the nonlinearity in asphalt concrete (c.f. Kose et al., 2000; Masad and Somadevan, 2002). Moreover, the evolution of micro-cracks and rate-dependent plastic hardening are other major sources of nonlinearity in the thermo-mechanical response of asphalt concrete. These experimental observations also clearly show that asphalt concrete exhibit rate-dependent recoverable deformation (viscoelasticity), rate-dependent irreversible deformation (viscoplasticity), and rate-dependent damage evolution...
(viscodamage). Furthermore, self-healing characteristics of the asphalt phase of asphalt concrete enables the asphalt concrete to undergo micro-damage healing under specific circumstances, such that the constitutive models that do not account for micro-damage healing of these materials significantly underestimate the fatigue life of these materials and therefore lead to a very conservative design of structural systems made of such materials. Therefore, a robust and comprehensive model for constitutive modeling of asphalt concrete should consider the couplings among temperature, viscoelastic, viscoplastic, viscodamage, and micro-damage healing models.

In the first part of this paper (Abu Al-Rub and Darabi, 2012) we presented a general thermodynamic-based model for constitutive modeling of time-, rate-, and temperature-dependent materials. We derived in addition to the classical macroforce balance; the viscoelastic, damage, and micro-damage healing macroforce balances directly from the principle of virtual power. Moreover, we employed the proposed framework to consider the couplings among temperature, viscoelastic, viscoplastic, damage, and micro-damage healing models. To show the capabilities of the proposed framework, we used the macroforce balances to derive the 3D general form of Schapery’s viscoelasticity model, Perzyna’s viscoplasticity model, the damage model proposed by Darabi et al. (2011), and the micro-damage healing model proposed by Abu Al-Rub et al. (2010). In this part of the study, we are concerned about the numerical implementation and verification of the highly nonlinear thermo-viscoelastic, thermo-viscoplastic, thermo-viscodamage, and thermo-healing models developed in the first part of this paper.

Schapery (1964) originally presented his equations in a set of differential equations and later expressed these equations using the hereditary integral form (Schapery, 1969) which is derived systematically in the first part of this paper. A detailed explanation of the existing strategies for implementation of Schapery-type viscoelastic models into the finite element codes has been recently presented by Crochon et al. (2010). These approaches in general require the storage of the variables, such as hereditary integrals, over the whole loading history which significantly increases the computational cost. To overcome this issue, several researchers have proposed incremental models for numerical implementation of the Schapery’s viscoelastic constitutive model, whether using original differential equations or using recursive methods for integrating the hereditary integrals. The incremental models relate the numerical solutions at subsequent time t only to the converged values at the end of the preceding time $t - \Delta t$. Introducing such incremental models significantly decreases the computational cost since they do not require the storage of the variables over the whole loading history, instead, the solution only depends on the converged variables at the preceding time $t - \Delta t$ (Taylor et al., 1970; Henriksen, 1984; Tuttle and Brinson, 1986; Lai and Bakker, 1996; Poon and Ahmad, 1999; Haj-Ali and Muliana, 2004). However, it should be noted that the number of quantities and variables from the previous time increment that should be stored increases, specifically once the constitutive model is generalized to the 3D cases. Therefore, a large number of variables (e.g., the hereditary integrals) should still be stored in order to numerically solve and update the solution at current time t. This has been a challenging issue in proposing more efficient numerical algorithms for finite element implementation of Schapery-type viscoelastic models that can reduce the computational cost. Recently, Crochon et al. (2010) proposed an implementation procedure based on the original differential equations and used the Finite Difference scheme to numerically solve the equations. They showed that their approach allows the use of higher-order finite difference schemes in addition to the first-order schemes in order to increase the convergence rate. However, their approach requires the determination of other sets of parameters associated with the introduced viscoelastic internal state variables in addition to the compliance/relaxation matrices. Also, the incremental form of their approach requires the storage of the introduced viscoelastic internal state variables at the preceding time $t - \Delta t$ to be used in solving the equations at current time t. Moreover, while using the second-order schemes increases the convergence rate, it requires the storage of the variables for the past two time increments in order to numerically update the results at the current time increment. In this work, we will use the recursive-iterative algorithm (Haj-Ali and Muliana, 2004) for numerical implementation of the Schapery-type nonlinear viscoelastic model derived in Part I of this paper. The iterative scheme is used to estimate the initial values for the stress-dependent variables in order to increase the convergence speed. Moreover, a higher convergence speed is also guaranteed by using a consistent tangent stiffness in the finite element code. The implemented viscoelastic model will then be coupled to the viscoplasticity, viscodamage, and healing models as will be described in the subsequent sections.

Perzyna viscoplastic model (1971) has been among the classical viscoplasticity theories that have been used to predict the permanent deformation of asphalt concrete (e.g., Seibi et al., 2001; Masad et al., 2005; Masad et al., 2007; Saadeh et al., 2007; Huang et al., 2011b). Two main approaches are available in the literature for numerical implementation of Perzyna-type viscoplastic model. In the first approach, the current stress state can be outside the yield surface, such that the yield function can have a positive value. In this case, the Kuhn–Tucker conditions are not applied. In this approach, the viscoplastic model is implemented by allowing the stress state to be outside the yield surface and directly define the relaxation equations in the stress space, such that the stress state returns to the yield surface as a function of time (e.g., Cormeau, 1975; Perzyna, 1996). The second approach is to include the contribution of the rate-dependent strain terms in the yield surface and define a rate-dependent yield surface (Wang et al., 1997). This approach has been proposed by Wang et al. (1997) and is referred to as consistency model. In this approach, the Kuhn–Tucker conditions are applied, and therefore the viscoplastic multiplier can be determined through the consistency condition. The defined rate-dependent yield surface can expand and shrink by hardening or softening effects as well as by hardening or softening rate effects (Voyiadjis et al., 2004; Abu Al-Rub and Voyiadjis, 2006; Voyiadjis and Abu Al-Rub, 2006). We adopt this approach, such that the viscoplastic equations can be solved very similar to rate-independent plasticity models. The evolution of the viscoplastic strain at each integration point will then be evaluated using the closest point projection method, also referred to as return mapping or radial return, which is derived from a
fully implicit Backward-Euler scheme (e.g. Simo and Hughes, 1998). Moreover, a consistent tangent stiffness for the coupled viscoelastic-viscoplastic constitutive model will be defined in order to decrease the computational cost.

However, including the effect of micro-damage evolution on the mechanical response of asphalt mixes is a crucial task that enables the constitutive model in capturing specific phenomenon such as fatigue damage, tertiary creep, and post-peak behavior of the stress–strain response. Several researchers have tried to couple the damage models to viscoelastic and/or viscoplastic models in order to simulate these phenomenon in asphalt concrete (c.f. Kim and Little, 1990; Park et al., 1996; Lee and Kim, 1998; Kim et al., 2007; Sullivan, 2008). Most of these models couple the Schapery’s damage model (Schapery, 1975b,a) to the viscoelastic and viscoplastic models. However, this model can only be used to predict viscoelasticity and damage in tensile loading conditions and the model also treats the material as a linear viscoelastic material irrespective of temperature and stress levels. On the other hand, models based on the continuum damage mechanics (CDM) have been used effectively in the literature to model the degradation in materials due to cracks and voids (e.g. Kachanov, 1958; Rabotnov, 1969; Lemaitre, 1992). One of the main reasons for the popularity of the models based on the continuum damage mechanics is that using the well-known concept of effective configuration and stress space substantially simplifies the implementation of the damage models and reduces the complexities associated with the direct coupling between the damage model and the rest of the constitutive model. The classical CDM framework is generalized in Part I of the paper to include micro-damage healing following the works of Abu Al-Rub et al. (2010) and Darabi et al. (2012).

Moreover, asphalt concrete pavements are subjected to the repeated loadings conditions where the rest periods are introduced between each loading cycle. As mentioned previously, some induced micro-damages could heal during the rest periods. This phenomenon makes the fatigue life of pavements longer. As we will show in this paper, using constitutive models without considering the micro-damage healing effects substantially underestimates the fatigue life of asphalt concrete pavements. Therefore, enhancing the model to consider the micro-damage healing effects is an inevitable ingredient of a robust and comprehensive constitutive model for asphalt concrete and for the materials that tend to heal (Wool and Oconnor, 1981; Schapery, 1989; Miao et al., 1995; Little and Bhasin, 2007; Abu Al-Rub et al., 2010; Voyiadis et al., 2011; Darabi et al., 2012). However, this important issue has not received enough attention in the literature.

Another challenging task is the numerical implementation of the micro-damage healing model and the procedure to couple it to the viscoelastic, viscoplastic, and viscodamage constitutive model. To alleviate this issue, we use the concept of the healing natural configuration (Abu Al-Rub et al., 2010; Darabi et al., 2012). This concept can be regarded as the extension of the fundamental basis of the continuum damage mechanics theories based on the effective stress concept (or equivalently based on the effective configuration, (Kachanov, 1958; Rabotnov, 1969)) to self-healing materials by introducing a physically-based natural healing configuration in order to link the traditional continuum damage mechanics theories to the continuum damage-healing mechanics theories. By following this approach, the proposed framework inherits the simplicity and robustness of the continuum damage mechanics theories (Kachanov, 1958; Rabotnov, 1969) and also makes it possible to apply the existing numerical techniques for the continuum damage theories to materials that tend to heal without demanding major modifications in the existing numerical algorithms. Moreover, the power equivalence hypothesis (e.g. Lee et al., 1985; Darabi et al., 2012) is postulated to numerically couple the damage and micro-damage healing models to the viscoelastic and viscoplastic models. Therefore, it is assumed that the power expenditures in the damaged and healing configurations are the same. This hypothesis is attractive for mechanisms associated with dissipation processes since the correct estimation of the dissipated energy is generally needed. It is noteworthy that using the power equivalence hypothesis along with the concept of the stress in the healing configuration is both numerically and physically interesting. Using the concept of the stress in the healing configuration eliminates numerical complexities associated with direct (i.e. explicit) coupling between the damage and healing constitutive equations and at the same time makes these simplifications physically sound since it allows the accurate estimation of the dissipated energy in the healing configuration.

Finding a systematic procedure for identifying the model parameters is another challenging issue in developing a proper constitutive model. The developed constitutive model should also be validated against extensive experimental data to ensure the model capabilities in predicting the mechanical response of materials under different loading conditions. Most of the developed constitutive models for asphalt concrete are, however, developed for specific cases and validated against specific types of tests and experimental data and are not applicable to predict the response of asphalt concrete in general multiaxial state of stress and at different temperatures (e.g. Park et al., 1996; Lee et al., 2000; Seibi et al., 2001; Gibson et al., 2003; Masad et al., 2005; Saadeh et al., 2007). Moreover, most of the available constitutive models for time- and rate-dependent materials and specifically asphalt concrete do not couple temperature, viscoelastic, viscoplastic, viscodamage, and micro-damage healing components and therefore cannot be used to predict the response of these materials for general cases (Park et al., 1996; Lee and Kim, 1998; Schapery, 1999; Gibson et al., 2003; Uzan, 2005; Kim et al., 2006a; Darabi et al., 2011; Huang et al., 2011a). In fact, very few studies are available that couple all these mechanisms to together to achieve a more general constitutive model (e.g. Abu Al-Rub et al., 2010).

Therefore, this work tries to contribute in closing this gap by presenting the numerical algorithms for the numerical implementation of the coupled thermo-viscoelastic, thermo-viscoplastic, thermo-viscodamage, and thermo-healing constitutive model presented in Part I of this paper. To avoid unnecessary complexities in numerical implementation, we use the concept of the healing configuration, proposed in the first part of this paper, to couple the viscoelastic and viscoplastic constitutive models to the damage and micro-damage healing models. Recursive-iterative and the consistent rate-dependent return mapping algorithms will be used to implement viscoelastic and viscoplastic models, respectively. Furthermore, we present a systematic procedure for the identification of the model parameters in a decoupled way which allows us to
determine the model parameters quite uniquely. We also validate the proposed model over an extensive set of experimental data, which has not been used in the calibration process, including creep, creep-recovery, triaxial, constant strain rate, and repeated creep-recovery tests over a range of temperatures, stress levels, confinement levels, and strain rates in both tension and compression. It should be mentioned that extensive testing protocol is required to fully calibrate the formulated multi-physics constitutive model for general cases. However, it should be noted that the presented constitutive model and numerical implementation have the flexibility of turning on/off specific parts of the model under specific conditions in order to reduce the computational cost and the required tests to calibrate the model. For example, the viscoplastic and micro-damage healing models can be turned off if the primary aim of the constitutive model is to predict the behavior at low temperature where viscoplasticity and healing are negligible. Designing more efficient experimental procedures to reduce the number of required tests for the model calibration will be the subject of a future work by the authors and their collaborators.

2. Constitutive model

In this section, we present the numerical integration of the thermo-viscoelastic–viscoplastic–viscodamage–healing model developed in the first part of this paper. Table 1 summarizes the key ingredients of the model which are used in this part to predict the mechanical response of asphalt concrete.

Table 2 summarizes the model parameters associated with the presented constitutive model and briefly describes the physical meaning of the parameters. The procedure for the identification of the model parameters will be discussed in detail in the subsequent sections.

Table 1

<table>
<thead>
<tr>
<th>The constitutive model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>– Additive strain decomposition and the healing configuration</td>
</tr>
<tr>
<td>( \sigma_t = [1 - \phi T \Delta t] \sigma_t ) (1)</td>
</tr>
<tr>
<td>( \varepsilon_t = \varepsilon_{tp} + \varepsilon_q, \quad \dot{\varepsilon}<em>t = \dot{\varepsilon}</em>{tp} + \dot{\varepsilon}_q ) (2)</td>
</tr>
<tr>
<td>( \dot{\varepsilon}<em>q = (1 - \phi) \dot{\varepsilon}<em>q, \quad \dot{\varepsilon}</em>{tp} = (1 - \phi) \dot{\varepsilon}</em>{tp}, \quad \phi = \phi(1 - T) ) (3)</td>
</tr>
<tr>
<td>– Thermo-viscoelastic model</td>
</tr>
<tr>
<td>( \dot{\varepsilon}<em>{tp} = g_0 D_0 \dot{\varepsilon}</em>{tp} \sigma_t + g_1 \varepsilon_{tp} ) (4)</td>
</tr>
<tr>
<td>( \Delta D(t) = \sum_{n=1}^{N} D_0</td>
</tr>
<tr>
<td>( \delta_{tp} = \exp\left[-\theta_0 \left(1 - \frac{T}{T_0}\right)\right], \quad \delta_{tp} = \exp\left[-\theta_1 \left(1 - \frac{T}{T_0}\right)\right] ) (6)</td>
</tr>
<tr>
<td>– Thermo-viscoplastic model</td>
</tr>
<tr>
<td>( \chi = \dot{\varepsilon}_{tp} - \beta_1 - \sqrt{2\lambda_3}</td>
</tr>
<tr>
<td>( \Psi = \dot{\varepsilon}_{tp} - \beta_1 ) (8)</td>
</tr>
<tr>
<td>( \varepsilon_{tp} = \frac{\sqrt{3} \gamma_{pl}}{2} \left[1 + \frac{1}{d_{pl}} + \left(1 - \frac{1}{d_{pl}}\right) \frac{3 \gamma_{pl}}{\sqrt{3} \gamma_{pl}}\right] ) (9)</td>
</tr>
<tr>
<td>( \delta_{tp} = \delta_{tp} \exp\left[-\theta_1 \left(1 - \frac{T}{T_0}\right)\right] ) (10)</td>
</tr>
<tr>
<td>– Thermo-viscodamage model</td>
</tr>
<tr>
<td>( \dot{\phi} = \frac{\Gamma_{val}}{\gamma_{val}} \exp(\kappa_{val}\phi) \delta_{pd} ) (11)</td>
</tr>
<tr>
<td>( \Psi = \dot{\varepsilon}<em>{val} - \beta_1 ); ( \dot{\varepsilon}</em>{val} = \frac{\sqrt{3} \gamma_{pl}}{2} \left[1 + \frac{1}{d_{pl}} + \left(1 - \frac{1}{d_{pl}}\right) \frac{3 \gamma_{pl}}{\sqrt{3} \gamma_{pl}}\right] ) (12)</td>
</tr>
<tr>
<td>( \delta_{pd} = \exp\left[-\theta_1 \left(1 - \frac{T}{T_0}\right)\right] ) (13)</td>
</tr>
<tr>
<td>– Thermo-healing model</td>
</tr>
<tr>
<td>( \dot{h} = \Gamma_{h}(1 - \phi)^{-1} (1 - h) \dot{h} ) (14)</td>
</tr>
<tr>
<td>( \delta_{h} = \exp\left[-\theta_1 \left(1 - \frac{T}{T_0}\right)\right] ) (15)</td>
</tr>
</tbody>
</table>

\( \sigma_t \) is the stress tensor, \( \varepsilon_t \) is the total strain tensor, \( \varepsilon_{tp} \) is the plastic strain tensor, \( \dot{\varepsilon}_t \) is the total strain rate tensor, \( \dot{\varepsilon}_{tp} \) is the plastic strain rate tensor, \( \phi \) is the healing parameter, \( \delta_{tp} \) is the plastic diaparameter, \( \beta_1 \) is the thermal parameter, \( \lambda_3 \) is the thermal parameter, \( \gamma_{pl} \) is the plastic parameter, \( \gamma_{val} \) is the valation parameter, \( \kappa_0 \) is the initial modulus parameter, \( \kappa_1 \) is the initial modulus parameter, \( \theta_0 \) is the thermal parameter, \( \theta_1 \) is the thermal parameter, \( \Gamma_{val} \) is the valation parameter, \( \Gamma_{h} \) is the healing parameter, and \( \gamma_{val} \) is the valation parameter.
can first update the stress tensor in the healing configuration, couplings of the damage and micro-damage healing models to the viscoelastic and viscoplastic models. In other words, we approach substantially simplifies the numerical implementation since it avoids the complexities associated with the direct calculation of the damage and micro-damage healing variables based on the obtained variables in the healing configuration. At this point, the trial strain increment in the healing configuration [Eq.(16)] will then be fed to the viscoelastic and viscoplastic constitutive models in order to update viscoelastic and viscoplastic state variables in the healing configuration. The next step is to calculate the damage and micro-damage healing variables which are functions of the stress in the healing configuration, and finally update the constitutive models in the damaged (nominal) configuration is known \( \Delta \sigma_{ij}^D \) which is not the same as the strain increment in the healing configuration \( \Delta \varepsilon_{ij}^D \). Hence, an iterative method is needed to obtain the strain increment in the healing configuration at current time \( t \). The increment of the total strain in the damaged configuration at current time \( t \), the values of the internal variables (\( \phi, h, \)) and the stress tensor at the previous increment \( t - \Delta t \) are known. The objective here is to update the current stress tensor and determine the current viscoelastic and viscoplastic strain increments. Hence, we start with a trial strain increment in the healing configuration, such that:

\[
\Delta \varepsilon_{ij}^{p2} = \left[ 1 - \phi^{e,-\Delta t} \left( 1 - h^{e,-\Delta t} \right) \right] \Delta \varepsilon_{ij}^e
\]  

(16)

The trial strain increment in the healing configuration [Eq. (16)] will then be fed to the viscoelastic and viscoplastic constitutive models in order to update viscoelastic and viscoplastic state variables in the healing configuration. The next step is to calculate the damage and micro-damage healing variables based on the obtained variables in the healing configuration. At the end of the analysis, the new and old values of the strain increments in the healing configuration will be compared in order to check convergence. However, in the following developments the superimposed “\( e \)” is removed from the strain increment in the healing configuration for simplicity purposes.

The total strain tensor, effective viscoplastic strain, and stress tensor in the healing configuration at the current time “\( t \)” can be written, respectively, as follows:

\[
\begin{align*}
\varepsilon_{ij}^e &= \varepsilon_{ij}^{e,0} + \Delta \varepsilon_{ij}^{e,-\Delta t} + \varepsilon_{ij}^{e,0,t,-\Delta t} + \Delta \varepsilon_{ij}^{e,0,t} + \Delta \varepsilon_{ij}^{e,0,t,p} \\
\bar{\varepsilon}_{ij}^p &= \bar{\varepsilon}_{ij}^{p,-\Delta t} + \Delta \bar{\varepsilon}_{ij}^p \\
\sigma_{ij}^e &= \sigma_{ij}^{e,0,-\Delta t} + \Delta \sigma_{ij}^e
\end{align*}
\]  

(17)

In the next subsections, the procedure for determination of \( \Delta \varepsilon_{ij}^{e,0,t,p} \), \( \Delta \varepsilon_{ij}^{e,0,t} \), \( \Delta \bar{\varepsilon}_{ij}^p \), and \( \Delta \sigma_{ij}^e \) will be presented. Moreover, in the following subsections, we consider the constitutive model at the reference temperature (i.e. \( T = T_0 \)), such that all temperature
coupling terms have the value of one. In this section, we first explain the procedure for the implementation of the viscoelastic model, then we couple it to viscoplasticity, and finally the coupled viscoelastic–viscoplastic model will be coupled to damage and micro-damage healing models.

2.1. Implementation of the viscoelastic model

The volumetric and deviatoric components of the viscoelastic strain tensor can be written using Eq. (4), such that:

\[
\ddot{e}^{\text{ref}} = \ddot{\varepsilon}^{\text{ref}} + \frac{1}{3} \ddot{\sigma}_{kk} \delta y = \frac{1}{2G} \ddot{\varepsilon}^{\text{ref}} + \frac{\ddot{\sigma}_{kk}}{9K} \delta y = \frac{1}{2} \ddot{s}_y + \frac{\ddot{S}}{3} \sigma_{kk} \delta y
\]

(20)

where \( \ddot{\varepsilon}^{\text{ref}} \) and \( \ddot{\sigma}_{kk} \) are the deviatoric strain tensor and the volumetric strain components, respectively, of the viscoelastic strain tensor. The parameters \( G \) and \( K \) are the shear and bulk moduli in the healing configuration, respectively; whereas \( J \) and \( B \) are the shear and bulk compliances in the healing configuration, which are related to the compliance modulus and Poisson’s ratio \( \nu \) (assuming constant \( \nu \) by:

\[
J = 2(1 + \nu)D; \quad B = 3(1 - 2\nu)D
\]

(21)

Moreover, \( S \) is the deviatoric stress tensor which can be written as follows:

\[
\ddot{S}_y = \sigma_y - \frac{1}{3} \sigma_{kk} \delta y
\]

(22)

where \( \sigma_y \), \( \sigma_{kk} \), and \( \delta y \) are the stress tensor in the healing configuration, the volumetric stress in the healing configuration, and the Kronecker delta, respectively. Deviatoric and volumetric components of the viscoelastic strain tensor can be rewritten as follows by using Eqs. (4) and (5) (Lai and Bakker, 1996; Haj-Ali and Muliana, 2004):

\[
\ddot{e}^{\text{ref}}_{y} = \frac{1}{2} \left[ g_1^1 \sigma_0 + g_1^2 \sigma_0^2 \sum_{n=1}^{N} J_n - g_1^1 \sigma_0^2 \sum_{n=1}^{N} J_n \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f} \right] S_y

- \frac{1}{2} \left[ g_1^1 \sum_{n=1}^{N} J_n \exp(-\lambda_n \Delta t^f) q_{ij,n}^{\Delta t^f} - g_1^2 \sum_{n=1}^{N} J_n \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f} S_y \right] \equiv \ddot{J} S_y - \ddot{d}_y
\]

(23)

\[
\ddot{e}^{\text{ref}}_{kk} = \frac{1}{3} \left[ g_1^1 \sigma_0 + g_1^2 \sigma_0^2 \sum_{n=1}^{N} B_n - g_1^1 \sigma_0^2 \sum_{n=1}^{N} B_n \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f} \right] \sigma_{kk}

- \frac{1}{3} \left[ g_1^1 \sum_{n=1}^{N} B_n \exp(-\lambda_n \Delta t^f) q_{ij,n}^{\Delta t^f} - g_1^2 \sum_{n=1}^{N} B_n \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f} \sigma_{kk} \right] \equiv \ddot{B} S_y - \ddot{V}_y
\]

(24)

where the variables \( q_{ij,n}^{\Delta t^f} \) and \( q_{ij,n}^{\Delta t^f} \) are the deviatoric and volumetric components of the hereditary integrals for each term \( n \) of the Prony series at the previous time step \( t - \Delta t \). The hereditary integrals are updated at the end of each converged time increment, which will be used for the next time increment, and are expressed as follows (Haj-Ali and Muliana, 2004):

\[
q_{ij,n}^{\Delta t^f} = \exp(-\lambda_n \Delta t^f) q_{ij,n}^{\Delta t^f} + \left( g_1^1 S_y + g_1^2 S_y \right) \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f}
\]

(25)

\[
q_{kk,n}^{\Delta t^f} = \exp(-\lambda_n \Delta t^f) q_{kk,n}^{\Delta t^f} + \left( g_1^1 \sigma_{kk} - g_1^2 \sigma_{kk} \right) \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f}
\]

(26)

\( \Delta t^f \) and \( \Delta t^f - \Delta t \) are time increments at times \( t \) and \( t - \Delta t \) respectively. The deviatoric and volumetric increments of the strain increment can be expressed using Eqs. (17), (23), and (24) as follows:

\[
\Delta \ddot{e}^{\text{ref}}_{y} = \ddot{e}^{\text{ref}}_{y} - \ddot{e}^{\text{ref}}_{y} = \ddot{J} S_y - \ddot{J} S_y - \frac{1}{2} \sum_{n=1}^{N} J_n \left[ g_1^1 \exp(-\lambda_n \Delta t^f) - g_1^2 \Delta t^f q_{ij,n}^{\Delta t^f} - \frac{1}{2} \Delta t^f \right]

\times \left[ \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f} \right] - g_1^1 \left[ \frac{1 - \exp(-\lambda_n \Delta t^f)}{\lambda_n \Delta t^f} \right]
\]

(27)

\[
\Delta \ddot{e}^{\text{ref}}_{kk} = \ddot{e}^{\text{ref}}_{kk} - \ddot{e}^{\text{ref}}_{kk} = \ddot{B} S_y - \ddot{B} S_y - \frac{1}{3} \sum_{n=1}^{N} B_n \left[ g_1^1 \exp(-\lambda_n \Delta t^f) - g_1^2 \Delta t^f q_{kk,n}^{\Delta t^f} \right] - \ddot{V}_y

- \frac{1}{3} \sum_{n=1}^{N} B_n \left[ g_1^1 \exp(-\lambda_n \Delta t^f) - g_1^2 \Delta t^f q_{kk,n}^{\Delta t^f} \right] \sigma_{kk}^{\Delta t^f}
\]

(28)
The problem in solving Eqs. (27) and (28) is that the nonlinear functions are not known at the current increment \( t \). Therefore, an iterative method can be used to find the correct stress state. Hence, Eqs. (27) and (28) are further linearized by assuming that \( \bar{g}_2 = g_2^{-\Delta t} \), such that the trial stress increment can be written as follows:

\[
\Delta \bar{\sigma}_y^{\text{tr}} = \frac{1}{J_{\text{tr}}} \left\{ \Delta \bar{\sigma}_y^{\text{tr}} + \Delta \bar{\sigma}_b^{\text{tr}} \right\}
\]

(29)

\[
\Delta \bar{\sigma}_{kk}^{\text{tr}} = \frac{1}{B_{\text{tr}}} \left\{ \Delta \bar{\sigma}_{kk}^{\text{tr}} + \Delta \bar{\sigma}_{kk}^{\text{tr}} \right\}
\]

(30)

where \( \bar{\sigma}_b^{\text{tr}} \) and \( \bar{\sigma}_{kk}^{\text{tr}} \) can be obtained using Eqs. (23) and (24) when the nonlinear parameters are functions of the trial stress. This study employs the iterative scheme to obtain the correct stress for a given strain increment. Before the onset of visco-plasticity, the residual strain will be defined as follows:

\[
R_y^{\text{new}} = \Delta \bar{\sigma}_y^{\text{new}} + \frac{1}{3} \Delta \bar{\sigma}_{kk}^{\text{new}} - \Delta \bar{\sigma}_y^{\text{tr}}
\]

(31)

The Newton–Raphson method will be used to minimize the strain residual in Eq. (31). Moreover, when the strain is totally visco-elastic, the program uses the consistent Jacobian matrix which is the consistent tangent compliance and is determined as:

\[
\mathbf{S}_y^{\text{eff}} = \frac{\partial R_y^{\text{new}}}{\partial \mathbf{\bar{\sigma}}_y} = \mathbf{J} \delta_k \delta_j + \frac{1}{3} \left( \bar{J} - \mathbf{J} \right) \mathbf{\delta}_{ij} \delta_{kl} + \sum_{n=1}^{N} \bar{B} [\exp(-\lambda_n \Delta t) \mathbf{q}_{ij,n}^{\Delta t} - g_2^{\Delta t} \left( 1 - \exp(-\lambda_n \Delta t) \right) \mathbf{\bar{\sigma}}_{ij}^{\Delta t}] \delta_{ij} - \frac{1}{3} \left( \bar{\mathbf{J}} - \mathbf{J} \right) \delta_{ij} \delta_{kl} \mathbf{\bar{\sigma}}_{ij}^{\Delta t} \delta_{kl}
\]

(32)

It should be noted that Eqs. (31) and (32) are not valid in the presence of visco-plastic deformations. These equations will be updated in the next subsection. Fig. 1 shows the flowchart for the implementation of the visco-elastic model. Note that, this flowchart is valid in the absence of visco-plasticity.

2.2. Implementation of the viscoplastic model

In the following, the numerical algorithm presented in Huang et al. (2011b) is recalled here and modified to include the effects of damage and micro-damage healing. We rewrite the viscoplasticity dynamic yield surface, presented in Eq. (7), as follows:

\[
\chi = \bar{\tau}^{\text{up}} - \beta \mathbf{t}_e - \sqrt{2/3}R(p, T) - \sqrt{2/3}C_3 \left( \frac{\dot{p}}{P^{\text{up}}} \right)^\frac{1}{r} \leq 0
\]

(33)

where \( R(p, T) \) is the isotropic hardening function and \( A \) is defined in Eq. (8).

The Kuhn–Tucker loading–unloading conditions are also valid for the dynamic yield surface \( \chi \), such that:

\[
\chi \leq 0: \quad \dot{p} \geq 0; \quad \dot{\mathbf{t}} = 0; \quad \chi = 0
\]

(34)

A trial dynamic yield surface can be defined using Eqs. (29), (30), and (33), such that:

\[
\chi' = \bar{\tau}^{\text{up}} - \beta \mathbf{t}_e' - \sqrt{2/3}R(p^{\Delta t}) - \sqrt{2/3}C_3 \left( \frac{\dot{p}^{\Delta t}}{P^{\text{up}}} \right)^\frac{1}{r}
\]

(35)

\( \Delta \dot{p}^{\Delta t} \) can be obtained by iteratively solving Eq. (35) using the Newton–Raphson scheme. The viscoplastic strain increment \( \Delta \bar{\sigma}_y^{\text{up}} \) can then be obtained by rearranging Eq. (33), such that:

\[
\Delta \bar{\sigma}_y^{\text{up}} = \sqrt[3]{2} \Delta \bar{\sigma}_y \mathbf{N}_y
\]

(36)

where \( \mathbf{N}_y = \frac{\mathbf{\bar{\sigma}}_y}{\sigma_0} \) is the unit direction of the viscoplastic strain tensor. In the Newton–Raphson scheme, the differential of \( \chi \) with respect to \( \Delta \dot{p} \) is needed, which can be expressed as follows:

\[
\frac{\partial \chi}{\partial \Delta \dot{p}} = \sqrt{2/3} \frac{\partial R}{\partial \Delta \dot{p}} \frac{1}{\Delta \dot{p} N} \left( \frac{\Delta \dot{p}}{P^{\text{up}}} \right)^\frac{1}{r}
\]

(37)
At the \(k+1\) iteration, the effective viscoplastic strain can be calculated as follows:

\[
\left( \frac{D}{C^{22}} p_t \right)^{k+1} = \left( \frac{D}{C^{22}} p_t \right)^k / C_0 \]

The recursive-iterative scheme presented in Section 2.1 for nonlinear viscoelasticity with the above Newton–Raphson algorithm is used to update the current stress, viscoelastic strain, and viscoplastic strain increments by minimizing the residual strain defined as:

\[
\bar{R}_{ij} = \Delta \varepsilon_{ij}^{ve} + \Delta \varepsilon_{ij}^{tp} - \Delta \varepsilon_{ij}^d
\]

The stress increment at the \(k+1\) iteration can be calculated by:

\[
\left( \Delta \sigma_{ij}^t \right)^{k+1} = \left( \Delta \sigma_{ij}^t \right)^k - \left[ \frac{\partial R_{ij}^t}{\partial \sigma_{kl}} \right] \left( R_{ij}^t \right)^k
\]

where the differential of \(R_{ij}^t\) gives the consistent tangent compliance, which is necessary for speeding numerical convergence and can be derived as follows:

\[
S_{ijkl} = \frac{\partial R_{ij}^t}{\partial \sigma_{kl}} = \frac{\partial \Delta \varepsilon_{ij}^{ve}}{\partial \sigma_{kl}} + \frac{\partial \Delta \varepsilon_{ij}^{tp}}{\partial \sigma_{kl}} = S_{ijkl}^{ve} + S_{ijkl}^{tp}
\]

where \(S_{ijkl}^{ve} = \frac{\partial \Delta \varepsilon_{ij}^{ve}}{\partial \sigma_{kl}}\) is the nonlinear viscoelastic tangent compliance which is presented in Eq. (32). However, the viscoplastic tangent compliance can be derived using Eqs. (35) and (36), such that:

\[
S_{ijkl}^{tp} = \frac{\partial \Delta \varepsilon_{ij}^{tp}}{\partial \sigma_{kl}} = \sqrt{\frac{2}{3}} N_{ij}^d \frac{\partial \sigma_{ij}}{\partial \sigma_{kl}} + \sqrt{\frac{2}{3}} \frac{\partial \sigma_{ij}}{\partial \sigma_{kl}} = 3 f \Delta \varepsilon_{ij}^{tp} \frac{N_{ij}}{2 C_3} \left( f \frac{f}{3 \sqrt{2/3 C_3}} \right)^{N-1} \frac{\partial \sigma_{ij}}{\partial \sigma_{kl}} + \sqrt{\frac{2}{3}} \frac{\partial \sigma_{ij}}{\partial \sigma_{kl}}
\]

where \(f = \gamma_p - \beta_1 - \sqrt{2/3 R}\) is the rate-independent yield surface. The tangent compliance can now be obtained by substituting Eqs. (32) and (42) into Eq. (41). The flowchart for implementing the coupled viscoelastic–viscoplastic model is presented in Fig. 2.
2.3. Implementation of the viscodamage and micro-damage healing models

Damage and micro-damage healing are formulated using the concept of healing configuration presented in the first part of this paper. The use of the healing configuration concept substantially simplifies the numerical implementation of damage and micro-damage healing models. The stress in the healing configuration can first be updated using the viscoelastic and viscoplastic models. The damage force which is expressed in terms of the quantities in the healing configuration can be calculated and used to calculate the damage rate. In this section, we show the implementation procedure for obtaining the damage density and healing variable at the current time $t$.

We can define the damage condition by rearranging Eq. (11), such that:

$$
\chi^{vd} = \left[ \frac{1}{Y_0} \right] Y \left[ \frac{1}{C_0} \right] \exp \left( \frac{k}{C_2} e_{eff} \right) - \frac{Y_0}{C_0} \Delta \phi^{1-\Delta t} = 0
$$

where $Y$ is the damage force and $\chi^{vd}$ is the damage loading condition. A trial value for viscodamage loading surface can be defined as:

$$
\chi^{vd,tr} = \left[ \frac{1}{Y_0} \right] Y \left[ \frac{1}{C_0} \right] \exp \left( \frac{k}{C_2} e_{eff} \right) - \frac{Y_0}{C_0} \Delta \phi^{1-\Delta t} = 0
$$

Fig. 2. The flow chart of the recursive-iterative Newton–Raphson algorithm for implementation of the coupled viscoelastic–viscoplastic model. Note that since damage is not present the strains and strain increments in the damaged (nominal) and healing configurations are the same.
Similar to viscoplasticity, the damage increment can be obtained using the Newton–Raphson scheme. However, it should be noted that the values of \( \mathbf{v} \) and \( \mathbf{v}_{\text{eff}} \) are constant during these trials, which substantially simplifies the implementation, since they are expressed in the healing configuration. However, the differential of the \( \chi^{\text{rd}} \), Eq. (43), with respect to \( \Delta \phi \) is needed which can be expressed as follows:

\[
\frac{\partial \chi^{\text{rd}}}{\partial \Delta \phi} = -q \left( \frac{\mathbf{v}}{Y_0} \right) \frac{\mathbf{v}(1 - \phi)}{Y_0} \exp(k\overline{e}_{\text{eff}}) - \frac{1}{\Gamma^{\text{rd}} \Delta t}
\]

Hence, the damage density increment at the \( k + 1 \) iteration can be obtained as follows:

\[
(\Delta \phi^i)^{k+1} = (\Delta \phi^i)^k - \left[ \left( \frac{\partial \chi^{\text{rd}}}{\partial \Delta \phi} \right)^k \right]^{-1} \chi^{\text{rd}}
\]

The damage density \( \phi \) can then be obtained, such that:

\[
\phi^i = \phi^i - \Delta \phi^i
\]

The same procedure can be applied to calculate the healing variable. To show that, we rewrite the healing evolution law, presented in Eq. (14), as follows:

\[
h = h^0_h(1 - \phi^i)^k(1 - h^i)^k
\]

In the first part of this paper, we defined the damage density and the healing variable as follows:

\[
\phi = A^D_t
\]

\[
h = A^h_t
\]

where \( A \) is the total cross sectional area; \( A^D \) is the total damage area including both healed and unhealed damages; and \( A^h \) is the area of the healed damages. Note that it is assumed that damage cannot occur during the healing process and vice versa. Hence, the healed area \( A^h \) remains constant during the damage evolution. However, during the healing process the total damaged area \( A^D \) which includes both healed and unhealed areas remain constant. Now let us consider the cases when damage evolves. In these cases the area of healed micro-cracks \( A^h \) increases. Hence, the dominator of Eq. (50) increase and, as a result, the healing variable decreases. In other words, during the damage evolution \( \phi \geq 0 \), such that:

\[
\Delta \phi^i = \phi^i - \phi^{i-\Delta t} = \frac{A^D_{t}}{A} - \frac{A^D_{t-\Delta t}}{A}
\]

On the other hand, the healing variable at time \( t \) can be written as:

\[
h^i = \frac{A^h_t}{A^D_t}
\]

Eq. (52) can be rewritten, such that:

\[
h^i = \left( \frac{A^h_t}{A^D_{t-\Delta t}} \right) \left( \frac{A^D_{t-\Delta t}}{A} \right) \left( \frac{A}{A^D_t} \right)
\]

However, since damage is evolving, the total area of the healed micro-cracks in the current increment is the same as that of the current increment, such that:

\[
A^h_{t-\Delta t} = A^h_t
\]

Substituting Eq. (54) into Eq. (53) and making use of Eqs. (49) and (50) yields:

\[
h^i = \frac{\phi^{i-\Delta t}}{\phi^i} h^{i-\Delta t}
\]

Eq. (55) represents the relationship between the current healing variable and the healing variable at the previous increment when damage evolves. However, during the rest periods the damage density remains constant and some micro-cracks heal. In this case the total damaged area \( A^D_t \) is constant while the area of healed micro-cracks may increase. Hence, the healing variable at the current time \( t \) can be written as follows:

\[
h^i = \frac{A^h_t}{A^D_t} = \frac{A^h_{t-\Delta t}}{A^D_{t-\Delta t}} + \frac{\Delta A^h_t}{A^D_{t-\Delta t}}
\]
However, during the healing process the total damaged area at time $t$ is equal to that at time $t - \Delta t$, such that:

$$A^{D,t} = A^{D,t-\Delta t}$$

(57)

Substituting Eq. (57) into Eq. (56) and making use of Eqs. (49) and (50) yields:

$$h^t = h^{t-\Delta t} + \dot{h}^t \Delta t$$

(58)

Hence, the healing variable should be updated as follows:

$$\begin{cases} h^t = \frac{\dot{\phi}^{t-\Delta t}}{\dot{\phi}} h^{t-\Delta t}; & \dot{\phi}^{t} \geq 0 \\ h^t = h^{t-\Delta t} + \dot{h}^t \Delta t; & \dot{\phi}^{t} = 0 \end{cases}$$

(59)

As mentioned before, at the beginning of the analysis the strain increment in the damaged (nominal) configuration is known $\Delta \varepsilon^v_0$, which is not the same as the strain increment in the healing configuration $\Delta \varepsilon^h_0$. Hence, an iterative scheme is needed to obtain the strain increment in the healing configuration at current time $t$. The total nominal strain increment at current time $t$ and the values of the internal variables (such as $\phi$ and $h$) at the previous increment $t - \Delta t$ are used for determination of the strain increment in the healing configuration [see Eq. (16)]. Hence, a convergence criterion is needed at the end of the analysis to compare the updated and the trial values of the strain increments in the healing configuration. Fig. 3 shows the flowchart for implementation of the coupled viscoelastic–viscoplastic–viscodamage–healing model.

The above formulated numerical algorithms are implemented in the well-known commercial finite element code Abaqus (2008) via the user material subroutine UMAT. The finite element model considered here is simply a three-dimensional single element (C3D8R) available in Abaqus.

Fig. 3. The flow chart for implementation of the coupled viscoelastic–viscoplastic–viscodamage–healing model.
The sensitivity analysis has shown that the number of elements will not affect the results obtained for numerical simulation of available experimental data. Therefore, the finite element model considered for these simulations is simply a three-dimensional single element (C3D8R) available in Abaqus. Moreover, to investigate the numerical stability of the presented numerical implementation, an asphalt concrete pavement layer subjected to a simplified uniform tire contact pressure is simulated for different mesh densities and time-increments. It is shown that the numerical results converge as the time increment size decreases and mesh density increases. These investigations are briefly presented in Appendix A.

3. Application of the model to asphalt concrete: model calibration

In this section, the presented thermo-viscoelastic-viscoplastic-viscodamage-healing constitutive model is calibrated using a set of experimental data on asphalt concrete tested at different stress levels, strain rates, and temperatures. The asphalt concrete used in this study is described as 10 mm Dense Bitumen Macadam (DBM) which is a continuously graded mixture with asphalt binder content of 5.5%. Granite aggregates and an asphalt binder with a penetration grade of 70/100 are used in preparing the asphalt mixtures. Cylindrical specimens with a diameter of 100 mm and a height of 100 mm are compacted using the gyratory compactor. Single uniaxial creep-recovery tests in compression at reference temperature are conducted to identify the viscoelastic and viscoplastic model parameters, whereas two creep tests that include the tertiary creep response at the reference temperature are conducted to identify the viscodamage model parameters. The micro-damage healing model parameters are also identified using the repeated creep-recovery test with rest period at the reference temperature. The temperature coupling term parameters are then identified by comparing the results at different temperatures. Moreover, the parameters that distinguish between compressive and extensive loading conditions (i.e. $d^{vP}$ and $d^{vE}$) are identified by comparing several tests in tension and compression. Finally, the identified model parameters are used to predict the mechanical response of asphalt concrete over an extensive experimental data including creep-recovery, creep, triaxial, constant strain rate test, and repeated creep-recovery tests over a range of temperatures, stress levels, confinement levels, loading-unloading times, and strain rates in both tension and compression. Tables 3 and 4 list the summary of the tests used for calibration and validation of the constitutive model, respectively.

The procedure for identification of the viscoelastic, viscoplastic, and viscodamage model parameters is presented by Darabi et al. (2011). They used the identified model parameters to predict the behavior of asphalt concrete in compression. However, their model predictions deviate from the experimental measurements for the repeated creep-recovery tests with different rest periods. In this paper, we will validate the model in both tension and compression and under different confinement levels. We will also show that including the micro-damage healing model substantially improves the prediction of repeated creep-recovery tests in both tension and compression.

3.1. The viscoelastic model parameters

The first step in the calibration process is to determine the viscoelastic model parameters at the reference temperature. To achieve this, the viscoelastic and viscoplastic responses in the recovery part of a single creep-recovery test should be separated. The advantage of conducting a creep-recovery test is that the viscoplastic strain during the recovery remains constant which makes it possible to separate the viscoelastic and viscoplastic strains. Fig. 4(a) shows a schematic single creep-recovery test in which the stress level $\sigma$ is kept constant up to time $t_a$ and is removed after time $t_o$. The strain response of the creep-recovery loading [Fig. 4(a)] is presented in Fig. 4(b).

Let us assume that the stress level is too low and/or the loading time is too short for the material to get damaged or, at least, the induced damage is negligible. Under this assumption, we can assume that the strains in the healing and nominal configurations are the same. The strain response at the end of the loading time $t_a$ can be additively decomposed into viscoelastic and viscoplastic components, such that:

$$\epsilon(t_a) = \epsilon^{vE}(t_a) + \epsilon^{vP}(t_a)$$

(60)

The same strain decomposition can be assumed for any time $t$ after the unloading time $t_a$ (i.e. $t > t_a$). However, the stress is zero during the recovery. Hence, the viscoplastic strain remains constant after the unloading time $t_a$ (i.e. $\epsilon^{vP}(t > t_a) = \epsilon^{vP}(t_a)$). Hence, we can write the following strain decomposition at point $t$:

Table 3

<table>
<thead>
<tr>
<th>Test</th>
<th>Stress level in kPa</th>
<th>Loading time (unloading time) in s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression Creep-recovery</td>
<td>2000</td>
<td>400</td>
</tr>
<tr>
<td>Compression Creep</td>
<td>1000, 1500, 2000</td>
<td>40, 30, 20</td>
</tr>
<tr>
<td>Compression Repeated creep-recovery</td>
<td>1500</td>
<td>60 (100)</td>
</tr>
<tr>
<td>Tension Creep</td>
<td>300, 500</td>
<td></td>
</tr>
</tbody>
</table>
Subtracting Eq. (61) from Eq. (60) and calculating $\Delta \tilde{\varepsilon}^e(t_a)$ and $\Delta \tilde{\varepsilon}^e(t)$ using Eqs. (4) and (5) yield:

$$\Delta \tilde{\varepsilon}^e(t_a) = \tilde{\varepsilon}(t_a) - \tilde{\varepsilon}(t) = g_0 D_0 \tilde{\sigma} + g_1 g_2 [\Delta D(t_a) - \Delta D(t - t_a)] \tilde{\sigma}; \quad t \geq t_a$$

A low stress level is applied in this creep-recovery test and, hence, the nonlinear viscoelastic parameters can be assumed to be one (i.e. $g_0 = g_1 = g_2 = 1$). Therefore, Eq. (62) can be simplified as follows for low stress levels:

$$\Delta \tilde{\varepsilon}^e(t_a) = \tilde{\varepsilon}(t_a) - \tilde{\varepsilon}(t) = D_0 \tilde{\sigma} + [\Delta D(t_a) - \Delta D(t - t_a)] \tilde{\sigma}; \quad t \geq t_a$$

$\Delta \tilde{\varepsilon}^e(t_a)$ can be calculated for each test data in the recovery region. Note that the right hand side of Eq. (63) is only a function of viscoelastic properties. Therefore, the linear viscoelastic model parameters $D_0$ and $\lambda_n$ (i.e. the Prony series coefficients, Eq. (5)) can be identified by minimizing the error between the experimental measurements for $\Delta \tilde{\varepsilon}^e$ and Eq. (63). Fig. 5 shows the separation of the viscoelastic and viscoplastic strains at the reference temperature when the applied stress is 1500 kPa and the loading time is 30 s.
3.2. The viscoplastic model parameters

The next step in the model calibration process is to identify the viscoplastic model parameters. Basically, the creep part of the analyzed creep-recovery test [Fig. 4] can be used in order to identify the viscoplastic model parameters at the reference temperature. In other words, the viscoplastic strain in the creep part can be obtained by subtracting the model prediction for the viscoelastic strain (using the viscoelastic model parameters obtained in the previous sub-section) from the total experimental measurements.

The dynamic viscoplastic yield surface in Eq. (7) for a uniaxial compression step-loading is expressed as:

\[ \chi = \sigma - \beta \frac{\sigma}{3} - \sqrt{2/3 |K_0 + K_1(1 - \exp(-K_2\bar{p})|)} - \sqrt{2/3C_3 (\frac{\Delta \rho}{\Delta t})^N} \approx 0 \]  

(64)

where \( \sigma \) is the applied uniaxial compressive stress. Rearranging Eq. (64) yields:

\[ \frac{\Delta \rho}{\Delta t} = \Gamma^{vp} \left( \frac{\sigma - \beta \frac{\sigma}{3} - \sqrt{2/3 |K_0 + K_1(1 - \exp(-K_2\bar{p})|)}}{\sqrt{2/3C_3}} \right)^N \]  

(65)

where \( \Delta \rho \) can be obtained using the separated viscoplastic strain in the creep region \( \Delta \varepsilon_1^{vp} \) using the following expression [Eq. (36)]:

\[ \Delta \rho = \sqrt{\frac{2}{3}} \frac{\Delta \varepsilon_1^{vp}}{N_{11}} = \sqrt{\frac{2}{3}} \frac{\Delta \varepsilon_{1t}^{vp}}{1 - \frac{1}{3}} \]  

(66)

Moreover, the effective viscoplastic strain \( \Delta \rho = \sqrt{2\Delta \varepsilon_0^{vp}\Delta \varepsilon_0^{vp}/3} \) for the uniaxial compression can be calculated as follows:

\[ \Delta \rho = \sqrt{\frac{2}{3}} \sqrt{(\Delta \varepsilon_{1t}^{vp})^2 + 2(\Delta \varepsilon_{2t}^{vp})^2} \]  

(67)

where \( \Delta \varepsilon_{1t}^{vp} \) and \( \Delta \varepsilon_{2t}^{vp} \) are the axial and radial viscoplastic strain increments, respectively. However, the available experimental data does not include \( \Delta \varepsilon_{2t}^{vp} \). Hence, we calculate \( \Delta \varepsilon_{2t}^{vp} \) using Eq. (36), such that:

\[ \frac{\Delta \varepsilon_{2t}^{vp}}{\Delta \varepsilon_{1t}^{vp}} = \frac{-N_{22}}{N_{11}} = \frac{1.5 + x}{3 - x} \]  

(68)

Experimental observations on asphalt concrete have shown that parameters \( x \) and \( \beta \) which are related to the angle of friction in asphalt concrete change slightly for different asphalt concretes (Seibi et al., 2001; Masad et al., 2007). Therefore, these parameters are assumed to be constant in this work and their values are listed in Table 5. Fig. 6 shows the plot of the plastic strain increment over time increment versus time (i.e. \( \Delta \rho/\Delta t - t \)) extracted from creep part of the creep-recovery tests at multiple stress levels. The viscoplastic model parameters \( \Gamma^{vp}, N, K_0, K_1, \) and \( K_2 \) can be identified by minimizing the error between the measurements and Eq. (65) once \( \Delta \rho/\Delta t \) is calculated from the analyzed experimental data. It should be noted that this procedure should be performed at multiple stress levels in order to obtain proper values for the viscoplastic model parameters and to guarantee the uniqueness of these parameters. Otherwise, the identified model parameters do not yield...
The authors have already developed a test (i.e. repeated creep-recovery test at various stress levels) that can be used more effectively to identify the viscoplastic model parameters. The repeated creep-recovery test at various stress levels is conducted on a single specimen and the stress level increases as the number of loading cycles increases. This test provides the results at multiple stress levels and can be used more effectively to identify the viscoplastic model parameters instead of using the single creep-recovery test at different stress levels which require several specimens. However, the current experimental data does not include such test and therefore several creep-recovery tests at multiple stress levels have been used to identify the viscoplastic model parameters.

### 3.3. The viscodamage and micro-damage healing model parameters

The loading times in the creep-recovery tests conducted to identify the viscoelastic and viscoplastic model parameters are too short for the material to get damaged. However, in other tests such as the creep tests, the load will usually remain on the specimen until failure. The loading times in these tests are long enough for damage to evolve which causes the secondary and primary creep to develop. The viscodamage and micro-damage healing model parameters have been identified by minimizing the error between the experimentally extracted $\frac{\Delta p}{\Delta t}$ and Eq. (65). Several stress levels should be used in this step in order to obtain proper and unique viscoplastic model parameters.

![Plot of the increment of the plastic strain over time](image)

The identified viscoelastic–viscoplastic–viscodamage–healing model parameters at the reference temperature are shown in Table 5.

### Table 5
The identified viscoelastic–viscoplastic–viscodamage–healing model parameters at the reference temperature.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$ (s$^{-1}$)</td>
<td>10</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>$D_n$ (kPa$^{-1}$)</td>
<td>$1.98 \times 10^{-7}$</td>
<td>$1.48 \times 10^{-6}$</td>
<td>$6.56 \times 10^{-7}$</td>
<td>$1.43 \times 10^{-6}$</td>
<td>$2.74 \times 10^{-6}$</td>
</tr>
<tr>
<td>$D_0$ (kPa$^{-1}$)</td>
<td>$3.5 \times 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.15</td>
<td>0.3</td>
<td>$5 \times 10^{-4}$</td>
<td>3.63</td>
<td>35</td>
</tr>
<tr>
<td>$N$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_0$ (kPa)</td>
<td>610</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_1$ (kPa)</td>
<td>610</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_2$ (kPa)</td>
<td>215</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{\text{viscoplastic}}$ (s$^{-1}$)</td>
<td>$4 \times 10^{-5}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{\text{me}}$ (s$^{-1}$)</td>
<td>700</td>
<td>5</td>
<td>30</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>$r_{\text{viscodamage}}$ (s$^{-1}$)</td>
<td>$2.5 \times 10^{-3}$</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$f_{\text{healing}}$ (s$^{-1}$)</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$satisfactory model predictions for other tests, such that the identified parameters should be adjusted again by trial and error. The authors have already developed a test (i.e. repeated creep-recovery test at various stress levels) that can be used more effectively to identify the viscoplastic model parameters. The repeated creep-recovery test at various stress levels is conducted on a single specimen and the stress level increases as the number of loading cycles increases. This test provides the results at multiple stress levels and can be used more effectively to identify the viscoplastic model parameters instead of using the single creep-recovery test at different stress levels which require several specimens. However, the current experimental data does not include such test and therefore several creep-recovery tests at multiple stress levels have been used to identify the viscoplastic model parameters.
tertiary creep responses. We calibrate the damage model using the secondary and tertiary creep region in a creep test since these responses are mostly caused by damage. Moreover, during the creep loading, micro-damage healing is expected to be negligible as compared during rest periods. To calibrate the viscodamage model at the reference temperature (i.e., $T = 20$ °C), the identified viscoelastic and viscoplastic model parameters at the reference temperature are used to predict the creep tests. These predictions usually match for the initial response and start deviating from the experimental measurements in the secondary and tertiary creep regions. This deviation should be compensated for by using the viscodamage model.

At the reference temperature, the viscodamage temperature coupling term has the value of one [i.e., $\psi^{rd}(T_{ref}) = 1$] such that Eq. (11) simplifies as follows:

$$\dot{\sigma} = \psi^{rd} \left[ \frac{Y(1 - \dot{\phi})}{Y_0} \right]^q \exp(k_i^{eff}) \tag{69}$$

The first step in identifying the viscodamage model parameters is to select an arbitrary reference stress level. The reference damage force $Y_0$ can be calculated easily using Eq. (12) as $Y_0$. We express the damage evolution law of Eq. (69) in terms of the damage force in the damaged (nominal) configuration by making use of Eq. (1), such that:

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Viscoelastic, viscoplastic, and viscodamage temperature coupling term parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_0$</td>
<td>$h_1$</td>
</tr>
<tr>
<td>1.73</td>
<td>4.64</td>
</tr>
</tbody>
</table>

Separate viscoelastic response in the recovery part of the creep-recovery tests using Eq. (63).

Identify the Prony series coefficients $D_0$ and $\dot{\lambda}_0$ at the reference temperature using Eqs. (5) and (63).

Calculate $\Delta \bar{e}^{\nu}$, $\Delta \bar{e}^{\rho}$, and $\Delta \bar{p}$ from the creep part of the creep-recovery tests using Eqs. (66), (67), and (68).

Identify the viscoplastic model parameters at the reference temperature by minimizing the error between the experimental measurements and Eq. (65).

Identify $\Gamma^{rd}$ and $k$ from a creep test at the reference temperature and stress level using Eq. (71).

Identify viscodamage stress dependency parameter $q$ from a creep test at the reference temperature, when $\sigma \neq \sigma_{ref}$ using Eq. (69).

Identify healing model parameters at the reference temperature from repeated creep-recovery test with rest periods using Eq. (14).

Identify temperature coupling terms model parameters by comparing experimental data and model predictions at different temperatures using Eqs. (6), (10), and (13).

Identify $d^{\nu}$ and $d^{\rho}$ from two creep tests in tension using Eqs. (9) and (12).

Fig. 7. The procedure for identification of the constitutive model parameters.
\[
\dot{\phi} = \Gamma^{\text{rd}} \left[ \frac{Y}{Y_0} \right]^q \exp(k\varepsilon_{\text{eff}}) \]  

(70)

However, the stress in the damage configuration during the creep test is constant. Hence, at the reference stress level, the damage force \( Y \) is the damage force at the reference stress level. Therefore, at the reference stress level, Eq. (70) can be simplified further as:

\[
\dot{\phi} = \Gamma^{\text{rd}} \exp(k\varepsilon_{\text{eff}}) 
\]

(71)

Now, the damage fluidity parameter \( \Gamma^{\text{rd}} \) and the strain dependency parameter \( k \) can be identified using a creep test at the reference temperature and stress level. The viscodamage stress dependency parameter \( q \) can finally be identified by comparing the experimental results and model predictions for a creep at another stress level which is different from the reference stress level.

The identified viscoelastic, viscoplastic, viscodamage model parameters yield reasonable predictions for creep, creep-recovery, and constant strain rate tests. However, these model parameters substantially overestimate the strain as recorded in the experimental data for repeated creep-recovery tests with rest periods. As discussed earlier, this behavior is attributed to micro-damage healing. In other words, asphalt concrete heals and retrieve part of its strength during the rest

Fig. 8. Model predictions and experimental measurements for creep-recovery test in compression at 10 °C.

Fig. 9. Model predictions and experimental measurements for creep-recovery test in compression at 20 °C.
periods. Hence, we calibrate the micro-damage healing model by comparing the model predictions and experimental measurements using a repeated creep-recovery test. The identified model parameters at the reference temperature are listed in Table 5.

3.4. The temperature coupling term model parameters

The presented constitutive model has the advantage of predicting the responses at other temperatures using the temperature coupling terms. The procedure outlined in Sections 3.1–3.3 can be used to identify the model parameters at the reference temperature $T_{ref}$, and the model ability to predict the response at other temperatures is achieved using the viscoelastic, viscoplastic, and viscodamage temperature coupling terms. In this study, the same temperature coupling terms are assumed for both viscoelastic and viscoplastic models (i.e. $\eta^v = \eta^p$) as suggested by the experimental study of Schwartz et al. (2002) on asphalt concrete. Creep-recovery and creep tests at different temperatures are used to identify the viscoelastic–viscoplas-
tic and viscodamage temperature coupling terms, respectively. Moreover, we did not identify the micro-damage healing temperature coupling term parameter since the current experiments only include the repeated creep-recovery tests at the reference temperature. Table 6 lists the identified values for the temperature coupling terms.

3.5. The parameters distinguishing between loading modes

It is not convenient to introduce a different set of model parameters in tension and compression. Therefore, the presented constitutive model captures different responses in compression and extension loading conditions through the $d$ parameters [i.e. $d_{vp}$ in Eq. (9) and $d_{vd}$ in Eq. (12)]. To achieve this goal, the variable $\tau_{vp}$ in the viscoplastic yield surface is expressed in

![Diagram](image_url)

**Fig. 12.** Model predictions and experimental measurements for creep test in tension using the same identified model parameters: (a) $T = 10 \, ^{\circ}C$ and (b) $T = 20 \, ^{\circ}C$. Using $d_{vp}$ and $d_{vd}$ makes the model capable of predicting experimental measurements in tension. The model reasonably predicts the failure time.
terms of the second and the third deviatoric stress invariants (i.e. $J_2$ and $J_3$), $d^{vp}$ is a material parameter representing the sensitivity of yield behavior to the hydrostatic pressure $I_1$. This parameter can also be defined as the ratio of the yield strength in uniaxial tension to that in uniaxial compression (i.e. $d^{vp} = \sigma^t / \sigma^c$). To ensure the convexity of the yield surface $d^{vp}$ ranges between 0.78 and 1. Therefore, $d^{vp}$ values less than one indicates that the yield strength of the material in tension is less than that in compression (Dessouky, 2005). Similarly, $d^{vd}$ captures the different damage responses in tension and compression. In fact, $d^{vd}$ magnifies the damage force $\gamma$ in tensile loading modes. These two parameters can be obtained by comparing experimental measurements and model predictions in tension and compression. In this study, $d^{vp} = 0.78$ and $d^{vp} = 0.16$ are identified. Fig. 7 shows the flowchart for obtaining the model parameters in a systematic manner.

![Flowchart for obtaining the model parameters](image)

Fig. 13. Model predictions and experimental measurements for triaxial test in compression using the same identified model parameters at different confinement at $T = 35^\circ C$ when $\sigma_1 = 500$ kPa: (a) axial strain versus time and (b) radial strain versus time.
4. Application of the model to asphalt concrete: model validation

4.1. Single creep-recovery test

The identified model parameters listed in Tables 5 and 6 are used to validate the model against another set of experimental data listed in Table 4 which have not been used in the calibration process. Model predictions and experimental measurements for the creep-recovery tests at different temperatures, stress levels, and loading times (LT) are shown in Figs. 8–10. Figs. 8–10 show that model predictions at temperatures 10 °C, 20 °C, and 40 °C agree reasonably well with the experimental measurements. However, Fig. 10 shows that the model predictions overestimate the experimental measurements.

Fig. 14. Model predictions and experimental measurements for triaxial test in compression using the same identified model parameters at different confinement at $T = 35$ °C when $\sigma_1 = 750$ kPa: (a) axial strain versus time and (b) radial strain versus time.
at temperature 40 °C. We believe that more experimental data at high temperatures is required to identify the viscoplasticity temperature coupling terms more accurately.

4.2. Single creep test

Model predictions and experimental measurements for creep tests in compression at different temperatures and stress levels are shown in Fig. 11. Fig. 11 shows that the model reasonably predicts primary, secondary, and tertiary creep responses in compression over a range of temperatures and stress levels. Interestingly, the model yields quite accurate predictions for...
the failure time which changes drastically from thousands of seconds to couple of seconds as the stress level changes. The same model parameters are used to predict the creep data in tension which is shown in Fig. 12.

Figs. 11 and 12 clearly show that the model can reasonably predict the experimental data in both tension and compression. It should be noted again that the same model parameters listed in Tables 5 and 6 along with $d_v^{pp}$ and $d_v^{pd}$ parameters are used to predict experimental measurements at different temperatures and in both tension and compression.

4.3. Triaxial test

The model is validated against triaxial tests in compression to investigate the confinement effects on the mechanical response of asphalt concrete. The triaxial tests are available at a single temperature of 35 °C but at different confinement levels (i.e. $\sigma_3$ in Figs. 13–16) for different axial stress levels (i.e. $\sigma_1$ in Figs. 13–16). Fig. 13(a) and (b) show the model predictions and

\[ \sigma_1 = 330 \text{ kPa} \]
\[ \sigma_1 = 460 \text{ kPa} \]
\[ \sigma_1 = 560 \text{ kPa} \]

**Fig. 16.** Model predictions and experimental measurements for triaxial test in compression using the same identified model parameters at different confinement at $T = 35$ °C when $\sigma_1 = 1000$ kPa: (a) axial strain versus time and (b) radial strain versus time.
experimental measurements, respectively, for the axial and radial strains at different confinement levels when the axial stress is 500 kPa. Fig. 13(a) shows that the model is capable of predicting the confinement effects, such that the material can sustain the same creep stress for a longer time as the confinement level increases. However, Fig. 13(b) shows that the model predictions for the radial strain underestimates the experimental data. The high variability of the experimental data at high temperatures (i.e. 35 °C for these tests) and time-dependent Poisson's ratio that changes as the material deforms could be the reasons for these underestimations. However, more experimental investigations and model comparisons are required to draw a satisfactory conclusion for such deviations of the model predictions from the experimental data.

Fig. 14 compares the model predictions and experimental measurements for the triaxial tests in compression at different confinement levels when the axial stress is 750 kPa. Fig. 14(a) yields reasonable model predictions for the axial strain compared to experimental measurements. However, the model slightly underestimates the radial strain as shown in Fig. 14(b).
Again, the material sustains the stress for a longer time as the confinement level increases. Model predictions for the damage density at different confinement levels when the axial stress is 750 kPa are shown in Fig. 15. Fig. 15(a) shows that the damage grows with slower rate as the confinement level increases. This test clearly confirms that the damage model is sensitive to confinement level. Also, Fig. 15(b) shows the damage density versus strain for different confinement level when the axial stress is 750 kPa. This Figure shows that the material experiences less damage density at a constant strain level as the confinement level increases. This is another interesting feature of the presented damage model which is due to the selected damage force (i.e. modified Drucker–Prager) in the viscodamage evolution function. Fig. 16 shows model predictions for the triaxial test at different confinement levels when the axial stress is 1000 kPa. Fig. 16(a) and (b) shows that the model predictions agree well with experimental data for both axial and radial strains.

4.4. Monotonic uniaxial constant strain rate test

The monotonic uniaxial constant strain rate tests at different temperatures and strain rates are conducted to test the model capability in capturing time-, rate-, and temperature-dependent response of asphalt concrete. Fig. 17(a) shows the model predictions and experimental measurements at different temperatures and strain rates for uniaxial constant strain rate test.

The plot of the predicted damage density versus the creep strain for strain rate of 0.005 s\(^{-1}\) is shown in Fig. 17(b). Fig 17(b) shows that: (1) the induced damage for the same strain level is larger at low temperatures which agrees with the experimental observations; (2) although the model does not possess a damage threshold and the damage can evolve at anytime during the loading, the induced damage at low strain levels is negligible and close to zero; (3) damage density reaches to its maximum rate approximately near the peak point of the stress–strain diagram and decreases in the post peak region. Hence, one may consider the inflection point of the damage–strain diagram as the strain corresponds to the peak stress at the stress–strain diagram. This behavior is related to the presence of the history term \((1 - \phi)\) in the damage model; (4) The presence of the history term in the damage model cause the damage–strain diagram to have the S-like shape which is intuitively sound.

The same model parameters listed in Tables 5 and 6 are used to predict the constant strain rate test in tension which is shown in Fig. 18.

Fig. 18 shows that the model predictions and experimental measurements does not match very well. The reason could be due to the fact that the viscoelastic model does not distinguish between the loading modes in extension and compression state of stresses. The authors are currently investigating this issue.

4.5. Repeated creep-recovery test

The ultimate goal of this work is to develop a unified model to predict the behavior of asphalt concrete pavements during their service life. However, pavements are subjected to repeated loading during the service life where fatigue damage
Fig. 19. Model predictions and experimental measurements for the repeated creep-recovery test in compression at $T = 20^\circ C$ when $\sigma = 1500$ kPa: (a) creep strain; (b) effective damage density and (c) healing variable.
plays a critical role in failure of these pavements. In this part, the model will be used to predict the repeated creep-recovery tests in compression and tension with different loading times and rest periods. These tests are conducted at the reference temperature (i.e. $T = 20\,^\circ C$) where the applied stress levels are 1500 kPa in compression and 300 kPa in tension. Fig. 19(a) shows the model predictions and experimental measurements for repeated creep–recovery test in compression for different loading times (LT) and unloading times (UT). Fig. 19(a) clearly shows that the model predictions when micro-damage healing is included are significantly improved as compared to experimental data and the predictions when micro-damage healing is not considered. Fig. 19(b) shows the model predictions of the effective damage density [i.e. $\phi = \phi(1 - \psi)$] versus time. This figure shows that the effective damage density in the presence of the micro-damage healing is no longer irreversible. In other words, the effective damage density may decrease during the rest period which is accompanied with the recovery in the stiffness and strength during the rest period. Fig. 19(c) shows the evolution of the healing variable. This figure shows that the healing variable increases during the rest period and decreases during the loading time. Moreover, Fig. 19(c) shows that the rate of the increase in the healing variable decreases for large values of the damage density. This observation is due to the presence of the damage history term $[(1 - \phi)]$ in the healing evolution function [Eq. (71)]. This term is included in the healing evolution law since the micro-cracks are too large to heal for large values of the damage density.

Fig. 20 shows another creep-recovery test in compression when the loading time is 60 s and the unloading time is 1500 s. Fig. 20 shows that including the healing parameter substantially enhances the model predictions. Fig. 20 shows that the model predictions without micro-damage healing component deviates substantially when the unloading time increases.

The same model parameters are used to predict the repeated creep-recovery tests in tension. Fig. 20 shows the model predictions and experimental measurements for the creep–recovery tests in tension. Figs. 19–21 show that considering the effect of the micro-damage healing substantially improves the ability of the model in predicting the fatigue life of asphalt concrete in the presence of rest periods. These figures also show that the effect of micro-damage healing on the extension of the fatigue life is more pronounced when the total duration of resting period increases.

5. Conclusions

In this study, we presented the numerical algorithms for implementation of the thermodynamic-based thermo-viscoelastic–viscoplastic–viscodamage–healing model developed in the first part of this paper. We used the recursive-iterative and rate-dependent return mapping algorithms to implement the viscoelastic and viscoplastic models, respectively. Moreover, we presented the constitutive model in the healing configuration in the context of the continuum healing–damage mechanics which substantially simplifies the numerical implementation of the highly nonlinear constitutive model. We discussed that using the healing configuration along with postulating the power equivalence transformation hypothesis are both numerically and physically interesting. In other words, using the healing stress space eliminates numerical complexities associated with the direct couplings of the damage and healing models with the rest of the constitutive model; whereas, power equivalence hypothesis makes these simplifications physically sound since it allows the estimation of a positive definite dissipated energy in the healing configuration.

We implemented the presented numerical algorithms in the well-known commercial finite element code Abaqus (2008) via the user material subroutine UMAT. The implemented constitutive model is then used to predict the mechanical response
Fig. 21. Model predictions and experimental measurements for the repeated creep-recovery test in tension at $T = 20\, ^\circ\mathrm{C}$ when $\sigma = 300\, \mathrm{kPa}$. (a) LT = 60, UT = 50; (b) LT = 60, UT = 100; (c) LT = 60, UT = 1500.
of asphalt concrete over a wide range of experimental data. Moreover, we presented a systematic procedure for determination of the model parameters using several creep and creep-recovery tests.

Model predictions and experimental measurements show that the model predicts creep-recovery experimental data at temperatures 10 and 20 °C well. However, model predictions of the creep-recovery at high temperatures deviate from the experimental data which is attributed to lack of experimental measurements at high temperatures to identify the viscoplasticity temperature coupling term.

It is shown that the model predictions for the creep tests agree well with experimental measurements at different stress levels and temperatures. It is shown that the model is capable of predicting the primary, secondary, and tertiary creep regions quite accurately. Time of failure at different stress levels and temperatures is also captured reasonably.

We showed that the model predictions for the triaxial tests at different confinement and axial stress levels agree reasonably well with the experimental measurements on asphalt concrete. It is shown that the constitutive equations, specifically the damage model, are sensitive to confinement level which is an important issue in modeling of asphalt concrete and other type of geomaterials. In this paper, it is shown that the derived constitutive equations are used successfully to predict the behavior of asphaltic materials, which are one type of geomaterials that are characterized by several complex forms of nonlinearity (e.g. nonlinear viscoelasticity, viscoplasticity, viscodamage, healing) that are rarely found combined in any other type of geomaterials. The proposed constitutive model includes many phenomenological aspects that are common in many geomaterials such as non-associative plasticity, different behavior in tension and compression, coupling between plasticity and damage, loading-path dependency, deformation history effects, etc. Therefore, all these features and more are already incorporated into the proposed constitutive model allowing one to predict the behavior of various types of geomaterials through applying several simplified assumptions.

Furthermore, model predictions for the constant strain rate tests at different temperatures and strain rates clearly show the model capabilities in capturing mechanical response of asphalt concrete.

It is shown that the micro-damage healing model substantially enhances the model in predicting the fatigue life of the asphalt concrete in the presence of the rest period. This effect becomes very substantial when the total duration of the rest period increases. It is shown that the model with micro-damage healing component can reasonably predict well the experimental data for the creep-recovery tests with different loading and unloading times.

Finally, the same model parameters along with the $d^p$ and $d^g$ are used to predict the experimental measurements in tension. It is shown that the model can reasonably predict the creep, constant strain rate, and repeated creep-recovery tests in tension. However, model predictions and experimental data did not match well since the viscoelastic model does not distinguish between extension and contraction loading states.

The ultimate goal of this work is to develop a unified constitutive model to predict the response of asphalt concrete pavements during their service life. Hence, it seems necessary to investigate the effect of environmental factors such as aging and moisture-induced damage in addition to the viscoelastic, viscoplastic, viscodamage, and micro-damage healing effects. Particularly, the inclusion of the moisture-induced damage which is one of the most important factors in deteriorating the asphalt concrete pavements during their service life seems inevitable. Models that consider the microstructural information have been shown to be effective tools to study the effects of environmental factors such as aging and moisture on mechanical response of materials (Abu Al-Rub et al., in press; You et al., 2012). The focus of future works by the authors will be on simulating the thermo-chemo-hydro-mechanical response of asphalt concrete by considering the effect of the microstructure on mechanical response of asphalt concrete, coupling the mechanical response to the fluid flow as the main mechanism for the moisture-induced damage, and coupling the thermo-mechanical response of asphalt concrete to chemical reactions as the primary mechanism of the oxidative aging in asphalt concrete. These goals will be obtained by considering the microstructural information on the constitutive model and by coupling these multi-physic mechanisms in these materials.

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Appendix A. Mesh size and time increment size sensitivity analysis

This appendix briefly investigates the mesh size and time increment size sensitivity of the presented constitutive model for two simple problems. The experimental tests are conducted on cylindrical specimens with a diameter of 100 mm and a height of 100 mm. The creep test with a stress level of 1500 kPa and loading time of 30 s applied on an axisymmetric plate is therefore considered here to investigate the mesh density and time-increment sensitivity. The identified model parameters listed in Table 5 are used in these examples. Fig. A1(a) shows the viscoplastic strain response for different time increments.
The results presented in Fig. A1(a) are obtained using a single element. Fig. A1(a) clearly shows that the viscoplastic response converges when the time increment is less than 1 s. For smaller time increments the results are almost identical to the case when time increment is 1 s. Therefore, the time increment size of 1 s is considered for conducting the simulations in this paper.

Fig. A1(b) shows that increasing the mesh density does not change the results for this problem, such that the viscoplastic strain remains the same regardless of the mesh density. Therefore, a single element and a time increment of 1 s are used to

Fig. A1. Time increment size and mesh density sensitivity analysis for a creep test when the stress level is 1500 kPa and loading time is 30 s: (a) results converge when time increment is less than 1 s and (b) results do not depend on number of elements. Therefore, the simulations are conducted using a single element when the time increment size is 1 s.

1500 kPa
100 mm
500 mm

Fig. A2. Schematic representation of a single layer asphalt concrete layer subjected to uniform contact stress of 1500 kPa.
simulate the experimental tests in this paper. The reason that this problem does not depend on the mesh density is that the stresses and strains remain uniform through the specimen because of the assumed boundary conditions for the lab tests. However, mesh sensitivity analysis should be conducted when simulating a problem for which the stress and strains are not uniform throughout the model.

As mentioned before, the ultimate goal of the presented constitutive model is to use it in the performance predictions of asphalt concrete pavements subjected to various traffic loading conditions. Permanent deformation (better known as rutting) is among one of the most important distresses in the asphalt concrete pavements. Therefore, a two-dimensional (2D) 100 mm thick asphalt layer subjected to the uniform normal stress of 1500 kPa (Fig. A2) is simulated here in order to ensure the stability of the numerical implementation and to investigate the mesh sensitivity of the results.

Permanent deformation at the center of the load is considered as the variable to compare different results for different mesh densities. Fig. A3 clearly shows that the results converge to a single curve as the mesh size decreases. This shows that the solution for this problem is not mesh-dependent. As shown by Needleman (1988), Loret and Prevost (1990), and Wang et al. (1997) the material rate-dependence introduces a length scale into the initial value problem, such that it regularizes the results and eliminates the mesh-dependent results in case of viscoplastic localization. This is attributed to the presence of the fluidity parameters (i.e. \( C_{vp}, C_{vd}, C_h \)) in all parts of the presented constitutive model. However, more investigations are required to confirm this results for the current constitutive model.

Fig. A4 shows the contours of the viscoplastic strain for different mesh densities. As shown in Fig. A4 the size of the localized region as well as the maximum value of the viscoplastic strain converges to definite values as the mesh density increases. This confirms the mesh-independent results obtained using the presented constitutive model. Also, the maximum value of the viscoplastic strain occurs at the half-top portion of the asphalt concrete pavement which is consistent with experimental observations and previous simulation results (Kim et al., 2006b; Abu Al-Rub et al., in press).

It should be noted that using the iterative scheme (Eqs. (29) and (30)) and the consistent tangent stiffness (Eq. (32)) along with the recursive scheme for the finite element implementation of the viscoelastic model greatly increases the convergence speed and reduces the number of required iterations. Normally, large number of iterations are required for the recursive schemes if the initial guess and iteration is not close enough to the converged values. However, incorporating an accurate iteration scheme along with the consistent tangent stiffness reduces the number of required iterations since the initial guess is close to the converged values. The number of iterations for the presented constitutive model has been less than 2 when the damage density is small. However, this number increases as the damage density increases, such that for large values of the damage density the number of iterations increases to about six times. The reason is that large values of the damage density makes the constitutive model highly nonlinear, such that more iterations are required for the constitutive model to converge.
Fig. A4. Viscoplastic strain contour for different mesh densities. As the mesh density increases, the maximum value of the viscoplastic strain converges and the results are not mesh-dependent. The maximum value of the viscoplastic strain occurs at the half-top portion of asphalt concrete pavement which is consistent with experimental observations (Kim et al., 2006b; Abu Al-Rub et al., in press).

References


